CONSTRUCTION OF A SEMIGROUP AUTOMATON FROM A GIVEN (3,2)-SEMIGROUP AUTOMATON

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Abstract: The aim of the talk is to present two algorithms about a construction of a semigroup automaton from a given (3,2)-semigroup automaton. The first one introduces new states on the arrows of the given (3,2)-semigroup automaton, while the second examines the recognizable words from the (3,2)-semigroup automaton and then deletes the arrows not accessible from those words.

Keywords: (3,2)-semigroup, semigroup automata, (3,2)-semigroup automata, languages, (3,2)-languages

1. Introduction

Here we recall the necessary definitions and known results. From now on, let B be a nonempty set and let $(B,\cdot)$ be a semigroup, where $\cdot$ is a binary operation.

A semigroup automaton is a triple $(S,(B,\cdot),f)$, where S is a set, $(B,\cdot)$ is a semigroup, and $f : S \times B \to S$ is a map satisfying

$$f(f(s,x),y) = f(s,x \cdot y) \text{ for every } s \in S, \quad x, y \in B. \qquad (1)$$

The set S is called the set of **states** of $(S,(B,\cdot),f)$ and f is called the **transition function** of $(S,(B,\cdot),f)$.

A nonempty set B with the (3,2)-operation $\{\} : B^3 \to B^2$ is called **a (3,2)-semigroup** iff the following equality

$$\{\{xyz\}t\} = \{x\{yzt\}\} \qquad (2)$$

is an identity for every $x, y, z, t \in B$. It is denoted with the pair $(B,\{\,\})$.

{ }	
a a a	*(b,a)*
a a b	*(a,a)*
a b a	*(a,a)*
a b b	*(b,a)*
b a a	*(a,a)*
b a b	*(b,a)*
b b a	*(b,a)*
b b b	*(a,a)*

Table 1

Example 1: Let $B = \{a, b\}$. Then the (3,2)-semigroup $(B, \{\ \})$ is given by Table 1.

A (3,2)-semigroup automaton is a triple $(S, (B, \{\ \}), f)$, where S is a set, $(B, \{\ \})$ is a (3,2)-semigroup, and $f : S \times B^2 \to S \times B$ is a map satisfying

$$f(f(s, x, y), z) = f(s, \{xyz\}) \text{ for every } s \in S,\quad x, y, z \in B. \qquad (3)$$

The set S is called the set of **states** of $(S, (B, \{\ \}), f)$ and f is called the **transition function** of $(S, (B, \{\ \}), f)$.

The transition function f of a (3,2)-semigroup automaton $(S, (B, \{\ \}), f)$ can be given by a table or by a graph. When we examine a graph of a (3,2)-semigroup automaton, then the nodes of graph are the states, and the arrows of the graph are the pairs of letters.

Example 2: Let $(B, \{\ \})$ be a (3,2)-semigroup given by Table 1 from Example 1 and $S = \{s_0, s_1, s_2\}$. A (3,2)-semigroup automaton $(S, (B, \{\ \}), f)$ is given by the Table 2 and the analogy graph by Fig. 1.

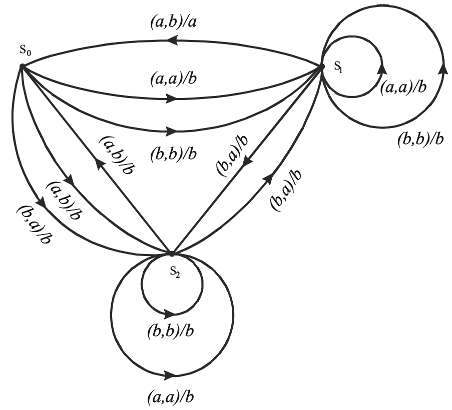

Figure 1

f	(a,a)	(a,b)	(b,a)	(b,b)
s_0	(s_1,b)	(s_2,b)	(s_2,b)	(s_1,b)
s_1	(s_1,b)	(s_0,a)	(s_2,b)	(s_1,b)
s_2	(s_2,b)	(s_0,b)	(s_1,b)	(s_2,b)

Table 2

Let $(Q,[\ ])$ be a free (3,2)-semigroup with a basis B constructed in (Dimovski, 1986).

Any subset $L^{(3,2)}$ of the universal language $Q^* = \bigcup_{p\geq 1} Q^p$, where $(Q,[\ ])$ is a free (3,2)-semigroup with a basis B is called a (3,2)-**language** on the alphabet B.

A (3,2)-language $L^{(3,2)} \subseteq Q^*$ is called **recognizable** if there exists:

(1) a (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$, where the set S is finite;

(2) an initial state $s_0 \in S$;

(3) a subset $T \subseteq S$; and

(4) a subset $C \subseteq B$,

such that

$$L^{(3,2)} = \{w \in Q^* \mid \overline{\varphi}(s_0,(w,1),(w,2)) \in T \times C\}, \qquad (4)$$

where $(S,(Q,[\ ]),\overline{\varphi})$ is the (3,2)-semigroup automaton constructed in (Dimovski, Manevska, 2001) for the (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$.

We also say that the (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$ **recognizes** $L^{(3,2)}$, or that $L^{(3,2)}$ is **recognized** by $(S,(B,\{\ \}),f)$.

Example 3: Let $(S,(B,\{\ \}),f)$ be a (3,2)-semigroup automaton given in Example 2. We construct the (3,2)-semigroup automaton $(S,(Q,[\ ]),\overline{\varphi})$ for the (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$. A (3,2)-language $L^{(3,2)}$, which is recognized by the (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$, with initial state s_0 and terminal state (s_2,b) is

$$L^{(3,2)}=\{w\in Q^* \mid w=w_1w_2...w_q,\quad q\geq 3,\ \text{ where }\ w_l=\begin{cases}(u_1^n,i),\ n\geq 3, u_\alpha\in Q\\ (a^*b^*)^*\end{cases},$$

$l\in\{1,2,...,q\}$, and:

a) If $i=1$, then:

a1) $(u_1^n,1)=a$, where $\psi_{p-1}(u_1)...\psi_{p-1}(u_n)=a^t b^j a^r b^h$ and

$$t+r=2k,\ \ t+j+r+h=n,\ \ t,j,r,h,k\in\{0,1,2,...\},\ \ k\geq 1;$$

a2) $(u_1^n,1)=b$, where $\psi_{p-1}(u_1)...\psi_{p-1}(u_n)=a^t b^j a^r b^h$ and

$$t+r=2k+1,\ \ t+j+r+h=n,\ \ t,j,r,h,k\in\{0,1,2,...\},\ \ k\geq 1;$$

b) If $i=2$, then $(u_1^n,2)=a$, where $\psi_{p-1}(u_1)...\psi_{p-1}(u_n)=(a^*b^*)^*$ and

$$\psi_p(w_1)...\psi_p(w_q)=b^*(ab^*)^{2k+1}\quad\}.$$

We will use a graphic presentation of (3,2)-semigroup automation in the next two algorithms.

1. First algorithm about construction of a semigroup automaton from a given (3,2)-semigroup automaton

Let $L^{(3,2)}$ be a (3,2)-language, which is recognized by the (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$, with an initial state s_0 and a set of terminal states $T\times C$, where $T\subseteq S$ and $C\subseteq B$.

By the definition of the (3,2)-language, $L^{(3,2)}$ is of the form (4). Now, we find the language $R=L^{(3,2)}\cap B^*$. It is of the form

$$R=\{w\in B^* \mid \overline{\varphi}(s_0,(w,1),(w,2))\in T\times C\}. \tag{5}$$

If $w \in R$ then $|w| \geq 2$ for all words in R, so we write $w = b_1 b_2 w'$, where $b_1, b_2 \in B$ and $w' \in B^*$.

We see that $f : S \times B^2 \to S \times B$ and the codomain of f is a subset of $S \times B$, so the transition from the initial state into another one is with the pair (b_1, b_2) into letter $b' \in B$. If t is the number of all pair $(b_1^{(j)}, b_2^{(j)})$ which goes from s_0 to the next states s_h and $b'^{(j)}$ for the words in R and $j \in \{1,2,...,t\}$, then we introduce new states $s_0^{(1,j)}$ on all the arrow, such that the transition function g on the semigroup automaton $(S,(B,\cdot),g)$ will be

$$g(s_0, b_1^{(j)}) = s_0^{(1,j)}, g(s_0^{(1,j)}, b_2^{(j)}) = s_h. \tag{6}$$

If $b_i^{(p)} = b_i^{(q)}$ for any $i \in \{1,2\}$, then $s_0^{(i-1,p)} \equiv s_0^{(i-1,q)}$, $s_0^{(i,p)} \equiv s_0^{(i,q)}$, where $s_0^{(0,l)} = s_0$ for $l \in \{1,...,t\}$.

If the new states is not defined for any $b_i^{(j)} \in B$, then we introduce a new state $\bar{s}_0^{(1,j)}$, such that $g(s_0^{(1,j)}, b_i^{(j)}) = \bar{s}_0^{(1,j)}$ and $g(\bar{s}_0^{(1,j)}, b) = \bar{s}_0^{(1,j)}$ for every $b \in B$.

In this way, one new state on each arrow which comes out from the initial state s_0 and which is on the path of the words in R is introduced. At the same time, a transition function g of the semigroup automaton which recognizes the language R is defined. This is a first step about transits from the initial state s_0 to the next state.

The codomain of f is a subset of $S \times B$, so in order to transit in each next state (s_j, b^j) for $b^j \in B$, $j \in \{2,...j_1\}$, where j_1 is the number of states in S through the words of R is gone, we take the next letter a^j of the word and go in the defined state (s_h, b^h) of the (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$ e.g

$$g(s_j, a^j) = f(s_j, b^j, a^j) = (s_h, b^h) \ . \tag{7}$$

The procedure continues unless we come to a terminal state and we reach all words of R. It means that the transition function g of the semigroup automaton $(S',(B,\cdot),g)$ is the same at the transition function f of the (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$, for all the states which through the word of R passes and are different from s_0.

The sets S and B are finite, so $S \times B^2$ and $S \times B$ are finite. It means that the graph of (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$ is finite, i.e. we introduce the paths of the words of R after a finite number of states or after a finite number of transition arrows from s_0 to the terminal states of $T \times C$. The language R may contain words with infinite length, but the (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$ is finite, so we will have same cycles after a finite number of steps to describe the words with infinite length.

Algorithm:

Step 1: On each arrow $(b_1^{(j)},b_2^{(j)})$ from the state $s_k \in S$ to the next states s_h introduce a new state $s_k^{(1,j)}$, $j \in \{1,2,...,t\}$, such that

$$g(s_k,b_1^{(j)}) = s_k^{(1,j)},\ g(s_k^{(1,j)},b_2^{(j)}) = s_h. \quad (8)$$

Step 1.1 If $b_i^{(p)} = b_i^{(q)}$ for any $i \in \{1,2\}$, $p,q \in \{1,2,...,t\}$, then

$$s_k^{(i-1,p)} \equiv s_k^{(i-1,q)},\ s_k^{(i,p)} \equiv s_k^{(i,q)}, \text{ where } s_k^{(0,l)} = s_k \text{ for } l \in \{1,...,t\}. \quad (9)$$

Step 1.2 If the new states are not defined for any $b_i^{(j)} \in B$, then introduce a new state $\bar{s}_k^{(1,j)}$, such that

$$g(s_k^{(1,j)},b_i^{(j)}) = \bar{s}_k^{(1,j)},\ g(\bar{s}_k^{(1,j)},b) = \bar{s}_k^{(1,j)} \text{ for every } b \in B. \quad (10)$$

Step 2: Repeat the procedure from Step 1 for each state of S unless the terminal state is reached all the words of R are included.

Example 4: Let $L^{(3,2)}$ be a $(3,2)$-language given in the Example 3. We examine the language $R = L^{(3,2)} \cap B^*$. It is of the form $R = \bigcup_{k=0}^{\infty}\{b^*\underbrace{ab^*ab^*...ab^*}_{2k+1}\}$. We construct the semigroup automaton $(S',(B,\cdot),g)$, which recognizes the language R, where the transition function g is given by the Table 3 and the analogy graph by Fig. 2.

g	a	b
s_0	$s_0^{(1,1)}$	$s_0^{(1,2)}$
$s_0^{(1,1)}$	s_1	s_2
$s_0^{(1,2)}$	s_2	s_1
s_1	$s_1^{(1,1)}$	$s_1^{(1,2)}$
$s_1^{(1,1)}$	s_1	s_0
$s_1^{(1,2)}$	s_2	s_1
s_2	$s_2^{(1,1)}$	$s_2^{(1,2)}$
$s_2^{(1,1)}$	s_2	s_0
$s_2^{(1,2)}$	s_1	s_2

Table 3

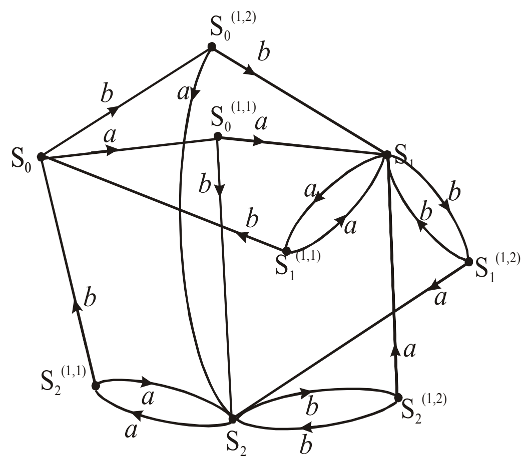

Figure 2

Second algorithm about construction of a semigroup automaton from a given (3,2)-semigroup automaton

Let $(S,(B,\{\ \}),f)$ be a (3,2)-semigroup automaton. The idea in the second algorithm is to construct semigroup automaton $(S\times B,(B,\cdot),\varphi')$ with a set of states $S\times B$ and a transition function φ' defined with $\varphi'(s,x,y)=f(s,x,y)$.

Let $L^{(3,2)}$ be a (3,2)-semigroup automaton recognized with the given (3,2)-semigroup automaton $(S,(B,\{\ \}),f)$ for initial state s_0 and set of terminal states

$T \times C \subseteq S \times B$. We find the language $R = L^{(3,2)} \cap B^*$ and we construct the semigroup automaton $(S \times B, (B, \cdot), \varphi')$, where the transition function $\varphi': S \times B \times B \to S \times B$ is defined with

$$\varphi'(s, x, y) = f(s, x, y). \qquad (11)$$

Now we find the language L' which is recognized with the semigroup automaton $(S \times B, (B, \cdot), \varphi')$ for initial states $\{s_0\} \times B$ and set of terminal states $T \times C$. Then we examine the words $w \in L' \cap R$ for initial state $\{s_0\} \times B$. If $\varphi'(s_0, b, w) \notin T \times C$ then $bw \notin R$, and if $\varphi'(s_0, b, w) \in T \times C$ then $bw \in R$. If $\varphi'(s_0, b, w) \notin T \times C$ then we delete the initial states (s_0, b) from the graph of the semigroup automaton $(S \times B, (B, \cdot), \varphi')$ together with the arrows which come out from that initial states. For the other initial states, we examine the arrows and states which are included in the recognition of words w from the initial state (s_0, b), $b \in B$ to terminal state of the set $T \times C$, i.e. we examine the paths through which that words go. If all the output arrows on the states through which the paths on the words w go are included in those paths, then the semigroup automaton which recognizes the language R is constructed. If a word $w' \in L' \cap R$ exists, for which a state (s_i, b') exists such that an output arrow b'', which is not come out on the path of w' to the terminal state of the set $T \times C$, we introduce a new state (s_i', b'), such that

$$\varphi'(s_i, b', b'') = (s_i', b'),\ \varphi'(s_i', b', b''') = (s_i', b') \text{ for each } b''' \in B. \qquad (12)$$

We repeat this procedure unless we reach some terminal state from $T \times C$.

Algorithm:

Step 1: Construct the language $R = L^{(3,2)} \cap B^*$ for the (3,2)-semigroup automaton $(S, (B, \{\ \}), f)$

Step 2: Construct the semigroup automaton $(S \times B, (B, \cdot), \varphi')$, where the transition function φ' is defined with $\varphi'(s, x, y) = f(s, x, y)$ and find the language L' which is recognized with it, for initial states $\{s_0\} \times B$ and set of terminal states $T \times C$.

Step 3: Examine the words $w \in L' \cap R$ for different initial states $\{s_0\} \times B$:

Step 3.1 If $\varphi'(s_0, b, w) \in T \times C$, then $bw \in R$, so select that initial states;

Step 3.2 If $\varphi'(s_0, b, w) \notin T \times C$, then $bw \notin R$, so delete that initial state from the graph of the semigroup automaton together with the arrows which come out from it.

Step 4: Examine the arrows and states which are included in the recognition of the words w from the initial state (s_0, b), $b \in B$, for each $\varphi'(s_0, b, w) \in T \times C$ to the terminal state from the set $T \times C$, i.e. examine the paths through which those words go.

Step 4.1 If all output arrows of the states through which the paths go of the words w from the initial state (s_0, b) to any terminal state from the set $T \times C$ are included in that paths, then the semigroup automaton which recognizes the language R is constructed. Go on Step 6.

Step 4.2 If a word $w' \in L' \cap R$ exists on whose path states (s_i, b') exist, such that arrows b'' which are not included in the path for the word w' to the terminal state from the set $T \times C$ come out, then introduce a new state (s_i', b'), such that $\varphi'(s_i, b', b'') = (s_i', b')$ and $\varphi'(s_i', b', b''') = (s_i', b')$ for each $b''' \in B$.

Step 5: Delete all states which are not included in Step 3 or Step 4 together with arrows which come out from them.

Step 6: The obtained semigroup automaton $(S', (B, \cdot), \varphi')$, where $S' \subseteq S \times B$ recognizes the language R.

φ'	a	b
(s_0, a)	(s_1, b)	(s_2, b)
(s_0, b)	(s_2, b)	(s_1, b)
(s_1, a)	(s_1, b)	(s_0, a)
(s_1, b)	(s_2, b)	(s_1, b)
(s_2, a)	(s_2, b)	(s_0, b)
(s_2, b)	(s_1, b)	(s_2, b)

Table 4

Example 5: Let $L^{(3,2)}$ be a $(3,2)$-language given in the Example 3. We examine the language $R = L^{(3,2)} \cap B^*$ which is of the form $R = \bigcup_{k=0}^{\infty} \{b^* \underbrace{ab^* ab^* \ldots ab^*}_{2k+1}\}$.

We construct the semigroup automaton $(S \times B, (B, \cdot), \varphi')$ with transition function $\varphi'(s, x, y) = f(s, x, y)$ which is given by Table 4 and the analogy graph by Fig. 3.

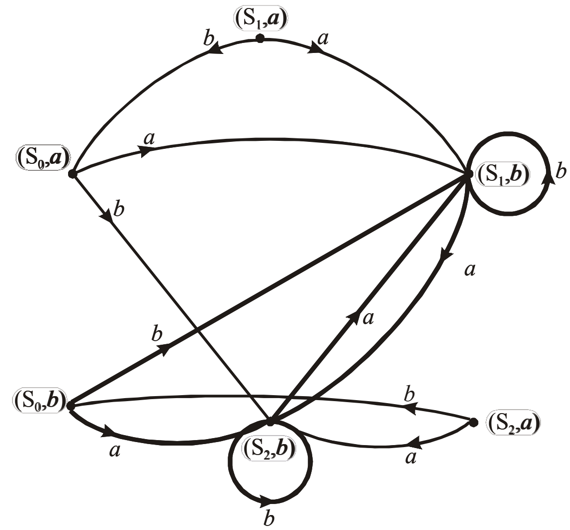

Figure 3

It recognizes the words $(b \cup ab^* a)(b^* ab^* ab^*)^*$ for a initial state (s_0, a)**,**while for initial state (s_0, b) it recognizes the words $(a \cup bb^* a)(b^* ab^* ab^*)^*$. i.e. it recognizes the language

$$L' = (a \cup bb^* a)(b^* ab^* ab^*)^* \cup (b \cup ab^* a)(b^* ab^* ab^*)^* . \qquad (13)$$

Now we examine the words $w \in L' \cap R$ and their paths. Because

$$L' \cap R = \{w \mid w = (a \cup bb^* a)(b^* ab^* ab^*)^*\}, \qquad (14)$$

the initial state of the semigroup automaton which recognized R is only (s_0, b).We examine the paths of the words which start in (s_0, b) and finish in (s_2, b), and we select them. Then we delete all states with their arrows which are not reached from the above initial state. In this way, we obtain the semigroup automaton $(S', (B, \cdot), \varphi')$ given by Table 5 and the analogy graph by Fig. 4, where $S' = \{(s_0, b), (s_1, b), (s_2, b)\}$.

φ'	a	b
(s_0, b)	(s_2, b)	(s_1, b)
(s_1, b)	(s_2, b)	(s_1, b)
(s_2, b)	(s_1, b)	(s_2, b)

Table 5

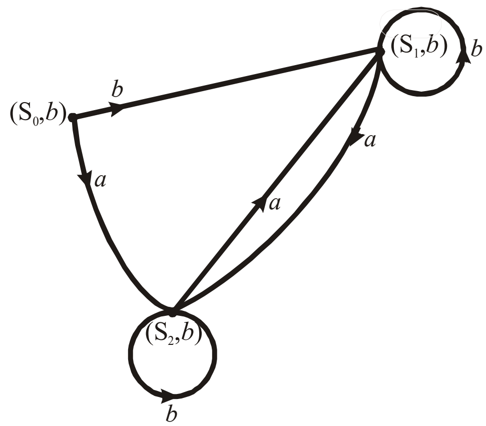

Figure 4

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