PERFORMANCE MEASURMENT MODEL OF PAY-TOLL SYSTEM

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ABSTRACT

In this paper, we try to model the natural behavior of traffic around pay-toll plazas. After that, we try to simplify traffic streams around pay-toll plazas by making few reasonable assumptions. We break-up the process in two stages: toll payment and lanes merging. In each stage we use Queuing Theory to model the queuing system. We derive a formula to calculate the average wasted time per driver. The average wasted time depends on the parameters of the system. By employing this formula, we calculate the average wasted time for different number of traffic lanes, traffic volume and number of pay-toll booths.

I. INTRODUCTION

Toll financing has been used for years for various purposes. Some of them include providing finances for modern freeways others for limiting vehicle flow in urban cores by encouraging transit usage. Despite its advantages, toll financing has many drawbacks which depend of the way the toll is collected. In the traditional stop-pay-go pay-toll systems used in Macedonia the biggest issue is time spending as the vehicle waits in line to be served, energy spending as the vehicle decreases its speed to 0 and increases it back to permitted freeway speed and pollution caused by unnecessary fuel combustion. The first logical solution for evading long lines hence decreasing waiting time is providing as many tollbooths as possible. This move causes another disadvantage; during heavy traffic, after toll payment, there is necessary time spending as the road decreases its width and the vehicles try to get back in line. Choosing the right number of tollbooths for getting an optimal balance between these two factors is crucial for best performance of these systems.

II. SYSTEM DEFINITION

Our system is based on the following assumptions:

- Constant traffic flow over a short time period
- The time between servicing two cars is of exponential distribution
- The traffic streams come into tollbooths evenly. We assume that exiting the tollbooth area does not contribute to delay. We ignore the delay caused by the behavior of irrational drivers.
- The drivers are delayed by waiting in lines for service. The driver has to wait for the other drivers that are in front of the line to be served first.
- The delay caused by waiting in lines is distributed exponentially.
- The drivers are delayed by the merging process after being serviced.
- The merging process is made with side merging (Fig.1).

Fig. 1: Side merging process

III. THE MODEL

According to our definition the delay in the system can be broken up in two parts: delay at the tollbooths, where drivers are waiting to be served, and delay at merging points, where drivers stop to wait to get into the merged lane. Here we employ Queuing Theory and consider each part as a queuing system. Based on the assumptions, the traffic stream coming from the entrance of the toll plaza is evenly divided into each tollbooth and each tollbooth will receive a stream whose inter-arrival time is of exponential distribution. The service time also has exponential distribution. Therefore, each tollbooth can be considered as an independent M/M/1 queue. Burke's Theorem states that the outcoming stream also has exponential distribution with the same rate as the arrival stream [1]. We will also use this property in the queue model of merging points.

The delay caused by the merging process is more complicated to analyze. We first considered a simple 2-to-1 merging process. When a driver on one lane arrives at the merging point, the delay time depends on whether there is a car on the other lane. For simplification, we treated the two incoming lanes as one queue. We suppose that every driver must stop and wait whenever there is another car in the queue. We define the service time of a car as the time it spends through the merging area. According to this, the service rate is equal to μ_B when there is more than one car in the system, and μ_0 when the system has only one car. Here, μ_B and μ_0 are constants that represent whether the driver does or does not have to give advantage to another car when merging. The service pattern of this queuing system is a general function. If the arrival pattern is exponential, the configuration of this queuing system would be M/G/1.

IV. MERGING PROCESS

The total merging process at the toll plaza is considered as a multiple 2-to-1 merging points. If there are T tollbooths that are merged back into N lanes, the number of merging points will be $T - N$. However, this depends on the arrival rate at the merging point which equals the traffic flow it receives. If a merging point receives a traffic stream coming from T tollbooths, with total flow of Φ, its arrival rate would be:

$$\lambda = \frac{k}{\mathrm{T}} \cdot \Phi \qquad (1)$$

The values of k for merging points depend on the merging layout. For example, in a toll plaza that uses side merging layout and T tollbooths, the first merging point takes the stream from 2 tollbooths, the second from 3 tollbooths etc.

The overall average wasted time is the weighted sum of all averaged wasted time at each merging point, where the corresponding weight is the probability for a driver to reach that point, which is k/T. The arrival rate and the corresponding probability at each merging point for a toll plaza with T tollbooths, N lanes at the exit that receives a total traffic flow Φ is displayed in Table 1.

Table 1: Arrival rate and probability at each merging point

Merging Point	1st	2nd	3rd	...	(T-N)th
Arrival Rate	$2\Phi/T$	$3\Phi/T$	$4\Phi/T$	...	(T-N+1)Φ/T
Probability	$2/T$	$3/T$	$4/T$	...	(T-N+1)$/T$

V. Calculation

Here, we will calculate the formula for wasted time depending on number of tollbooths. The calculation is based on some constants which must be defined first:

- Number of incoming lanes (N)

The common number of lanes on a highway (in one direction) in Macedonia is 2. In other countries, this number may be from 1 to 6.

- Total traffic flow (Φ)

The maximum traffic flow per lane is 2000/hr [2]. With N ranging from 1 to 6, various values for traffic flow will be considered, both for light and heavy traffic conditions.

- Service rate at the tollbooth (μ_A)

The service rate at a traditional tollbooth system where person disburses change, issues receipts, etc. is about 350 vehicles per hour [3].

- Service rate at merging point: when merging does not happens (μ_0)

This value represents the time for a vehicle to drive through the merging area at average highway speed. The average highway speed is 90km/h. The length of the merging area is the average car length plus the safety distance, which is [4] :

$$4{,}5\ [m] + 6 \cdot 4{,}5\ [m] = 31{,}5\ [m]$$

According to this, the average service time is:

$$31{,}5\ [m]\ /\ 90[km/h]\ = 1{,}26\ [1/s]$$

And the service rate:

$$\mu_0 = 3600/1{,}26\ =\ 2857\ [1/h].$$

- Service rate at merging point: when merging happens (μ_B)

To estimate μ_B we use the same approach we used to estimate μ_0. We considered that a vehicle passes the same distance with initial speed of zero. On this speed, the safe distance is one car length, and the average acceleration of a vehicle is 1.98 [m/s2]. Therefore the average service time is:

$$\sqrt{(2 \cdot (4{,}5 + 4{,}5)/1.98)} = 3.015[s]$$

And the service rate:

$$\mu_B\ =\ 3600\ /\ 3.015\ =\ 1194\ [1/h].$$

VI. Wasted Time

We divided the total wasted time in two parts: wasted time at the tollbooth and wasted time at the merging point.

The average wasted time of each tollbooth can be derived using the performance measure formula [1], given the arrival rate (Φ/T) and service rate (μ_A) is:

$$w_A = \frac{1}{\mu_A - \Phi/T} \qquad (2)$$

The merging point is modeled as a Markov system (Fig.2). Here, each state represents the number of vehicles in the system. The single μ_0 represents the possibility of no conflict at the merging point. The arrival rate is represented with λ. We first need to find the average waiting time in the system $t_{sys}(\lambda)$ in order to calculate the average wasted time $t_{diff}(\lambda)$.

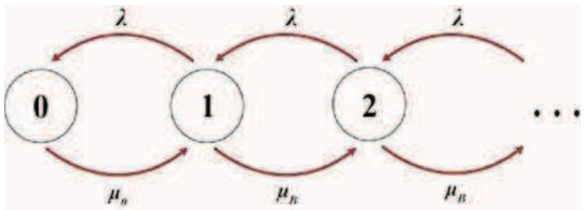

Fig. 2: State Diagram Representing the Queue at Merging Point

Let P_n be the probability that there are n drivers in the system. When the system reaches equilibrium, the net probability of transition is zero for each state. The sum of all P_n must be one.

$$\begin{cases} \lambda P_0 = \mu_0 P_1 \\ \lambda P_1 + \mu_0 P_1 = \lambda P_0 + \mu_B P_2 \\ \lambda P_n + \mu_B P_n = \lambda P_{n-1} + \mu_B P_{n+1}, n \geq 2 \\ \sum_0^{\infty} P_i = 1 \end{cases} \qquad (3)$$

The expected numbers of drivers in the system is:

$$L(\lambda) = \sum_0^{\infty} iP_i = \frac{\lambda}{\mu_B - \lambda} + \frac{\lambda(\mu_B - \mu_0)}{\lambda(\mu_B - \mu_0) + \mu_0\mu_B} \qquad (4)$$

By employing Little's law we obtained:

$$t_{sys}(\lambda) = \frac{L(\lambda)}{\lambda} = \frac{1}{\mu_B - \lambda} + \frac{\mu_B - \mu_0}{\lambda(\mu_B - \mu_0) + \mu_0\mu_B} \qquad (5)$$

The average wasted time of a driver at a merging point is the difference between t_{sys} and the time the driver would spend on a normal lane. The time a driver spends when no merging occurs is $\frac{1}{\mu_0}$.

$$t_{diff}(\lambda) = t_{sys}(\lambda) - \frac{1}{\mu_0} = \frac{1}{\mu_B - \lambda} + \frac{\mu_B - \mu_0}{\lambda(\mu_B - \mu_0) + \mu_0 \mu_B} - \frac{1}{\mu_0} \quad (6)$$

According to Table I, we calculated the corresponding probabilities for reaching the merging points that are 2/T, 3/T, 4/T or: $\frac{T-N+1}{T}$. Finally, the wasted time during the complete merging process is the weighted sum:

$$w_B = \sum_{i=1}^{T-N} \frac{i+1}{T} \cdot t_{diff}\left(\frac{i+1}{T} \cdot \Phi\right) \quad (7)$$

From Eqs. (2) and (7), we obtain the final equation for total wasted time in toll plaza:

$$w = w_A + w_B = \frac{1}{\mu_A - \Phi/T} + \sum_{i=1}^{T-N} \frac{i+1}{T} \cdot t_{diff}\left(\frac{i+1}{T} \cdot \Phi\right) \quad (8)$$

The total wasted time takes part of the total travel time through the toll plaza, so by minimizing it, the travelling time is also decreasing.

VII. Application

The Eq. (8) depends by the number of lanes N, number of tollbooths T and the traffic flow Φ. By applying this equation for the specific case on the 2 lane highway between Gostivar and Tetovo (Republic of Macedonia), with 4 maximum available tollbooths we obtain the results in Fig. (3) and Fig.(4) which are very close to the results obtained by measurements conducted on the field.

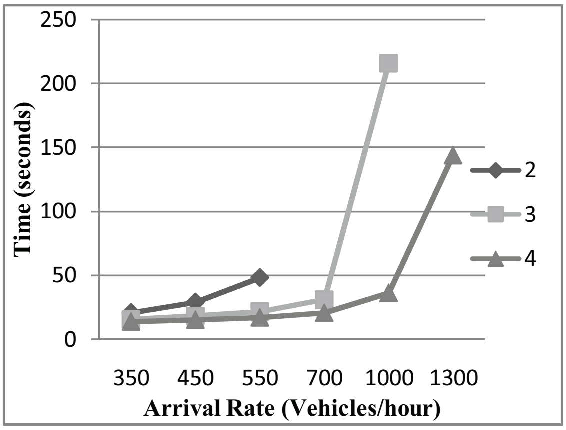

Fig. 3: Waiting time at a tollbooth for toll-plaza with 2, 3 and 4 tolbooths

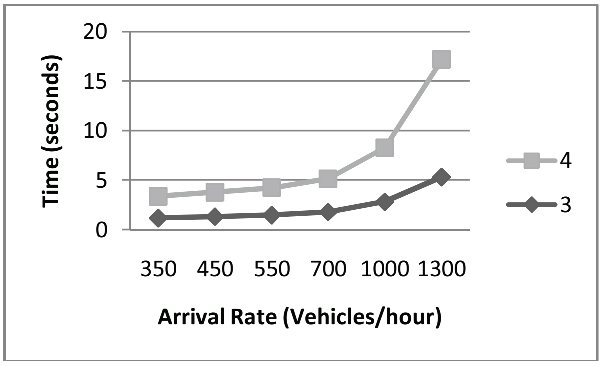

Fig. 4: Waiting time at merging point for a toll plaza with 3 and 4 tollbooths

It is obvious that, as the traffic intensity increases, the waiting time at tollbooth is decreased by activating more tollbooths at the same time. On the other side, with this action, the waiting time at merging point increases. The results obtained by employing this model for highways with more lanes and more tollbooths are more drastic.

References

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