PROPAGATION OF NONLINEAR SEISMIC WAVES IN SEMIBOUNDED 1- D MEDIA: A NUMERICAL APPROACH

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ABSTRACT

Two problems of vertical propagation of seismic SH wave through nonlinear media is considered. The seismic wave is half sine pulse-like wave, prescribed in the soil.

First, the problem of single layer (or one-story building) sitting on a half-space is considered. The response is studied on 1-D numerical model of uniform shear beam. The response of the layer depends upon:

- excitation properties (the amplitude and the duration of the prescribed input half-sine pulse),
- layer's nonlinear properties (the yielding strain and the nonlinear modulus),
- the ratio of impedances of layer and half-space.

Second, the response of a recorded building inVan Nuys, Los Angeles metropolitan area, is considered. The response is studied on 1-D multi-layered shear beam model. Although the 1-D model is the simplest numerical model, it gives excellent agreement with the recorded response.

I. INTRODUCTION

Rational design of earthquake-resistant structures requires realistic representation of the problem—that is, wise selection of the mathematical model and the associated differential equations. Once the model and the governing equations have been selected, the method of solution can also influence how realistic the end result will be. In the traditional earthquake engineering, structures have been represented both by lumped mass and by continuous models. The methods of the solution were usually based on the vibrational approach, using the superposition of modal responses.

The vibrational approach for the solution of the response of structures was formulated by Biot in early 1930s [1]. In this approach, the linear response is represented by superposition of the responses of the equivalent degrees of freedom, corresponding to the responses for the generalized coordinates, whose amplitudes in time, determined by excitation, are expressed via the Duhamel's integral [7]. It can be shown that for the linear systems such a representation is complete.

Analyzing structures using wave propagation models have been used for many years [5] but are only recently beginning to be verified against observations [3, 6].

In this paper, I will describe the most elementary form of the one-dimensional shear wave propagation in the structure with bilinear material properties. I will use finite differences to calibrate the response, and will focus mainly on the most elementary aspects and consequences of non-linear response. After a brief presentation on the method of computation, I will describe the results of two problems of one-dimensional wave propagation: in a single layer supported by a half-space, and in multi-layered medium again supported by a half space.

II. MODEL AND NUMERICAL SCHEME

The horizontal shear displacements, u, are cosidered in a 1-D model of a layer or one-story building, supported by a half space (Fig. 1a) and excited by a vertically propagating shear wave represented by a half-sine pulse (Fig. 1b). The Lax-Wendroff finite-difference scheme for solution of this problem with accuracy $O(\Delta t^2, \Delta x^2)$, where Δx and Δt are the space and time increments, leads to the exact solution when $\beta \Delta t / \Delta x = 1$, where β is the velocity of shear waves.

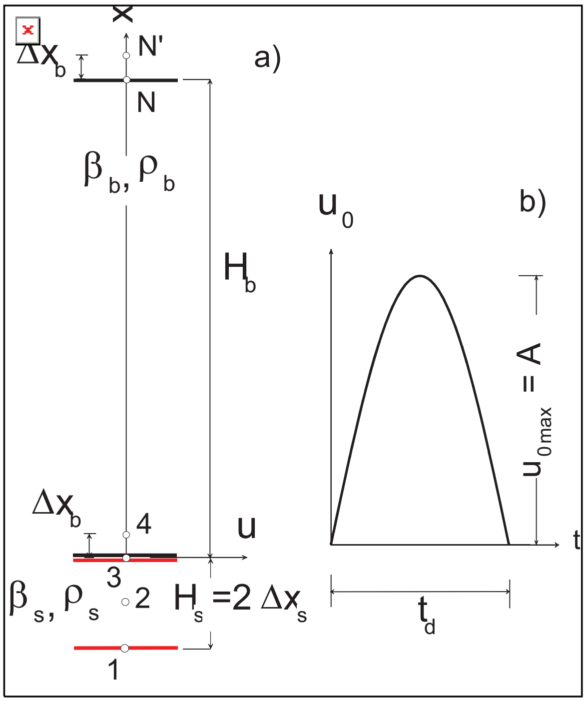

Fig. 1: Building and incoming strong-motion displacement pulse: (a) model of the building, and (b) the pulse in the half space.

With the ratio of the spatial intervals $\Delta x_b / \Delta x_s = \beta_b / \beta_s$, this requirement can be satisfied. The subscripts b and s

designate the values in the layer (or one-story building), with height H_b, and in the half space, respectively.

The equation of motion is

$$v_t = (\sigma)_x / \rho , \tag{1a}$$

and the relation between the derivative of the strain and the velocity is

$$\varepsilon_t = v_x , \tag{1b}$$

where v, ρ, σ, and ε are particle velocity, density, shear stress, and shear strain, respectively, and the subscripts t and x represent derivatives with respect to time and space.

The constitutive law $\sigma = \sigma(\varepsilon)$ is bilinear and is presented on Fig. 2.

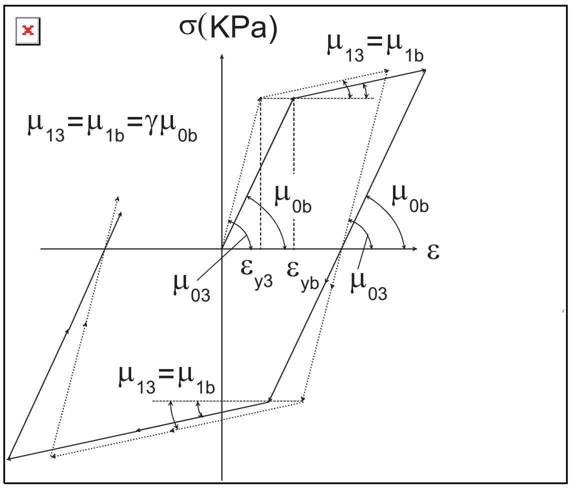

Fig. 2: The constitutive laws, $\sigma - \varepsilon$, for the building (solid line) and for the interface (dotted line).

Equations (1) can be written in vector form as

$$\frac{\partial \mathbf{U}}{\partial t} = \frac{\partial \mathbf{F}}{\partial x}, \tag{2}$$

where

$$\mathbf{U} = \begin{Bmatrix} v \\ \varepsilon \end{Bmatrix} \text{ and } \mathbf{F} = \begin{Bmatrix} \dfrac{\sigma}{\rho} \\ v \end{Bmatrix} = \begin{Bmatrix} \dfrac{\mu(\varepsilon)\cdot\varepsilon}{\rho} \\ v \end{Bmatrix}. \tag{3}$$

The boundary conditions (free stress at the top of the layer and continuity of stress and displacement at all of the interfaces and at point 3 in Fig. 1a) and the exact transmitting boundary condition in the soil (at point 2 in Fig. 1a) have been discussed in [2] and in [4].

It is assumed that the incoming wave is known and that its displacement as a function of t is prescribed at the point 1 ($x = -2\Delta x_s$). Also, in this paper it is assumed that the soil is always in the linear elastic state.

The marching-in-time procedure is established by approximating (2) by Lax-Wendroff numerical scheme.

III. NUMERICAL EXAMPLES

A. *Single Layer or one-story building*

As a numerical example, we consider a shear beam supported by elastic soil, as shown in Fig. 1a. The densities of the soil and of the beam are assumed to be the same: $\rho_b = \rho_s = \rho = 2000$ kg/m^3. The velocity of the shear waves in the soil is taken as $\beta_s = 250 m/s$, and in the building as $\beta_b = 100\, m/s$. To study non-linear response and the development of permanent deformations in the beam, we introduce two dimensionless parameters:

- dimensionless amplitude $\alpha = \dfrac{A}{H_b \cdot \varepsilon_{yb}}$, (4a)

  where: A is the amplitude of the pulse (see Fig. 1b), H_b is the height of the layer (building), and ε_{yb} is the yielding strain in the layer (building); and

- dimensionless frequency

$$\eta = \frac{H_b}{\dfrac{\lambda_b}{2}} = \frac{H_b}{\dfrac{\beta_b \cdot 2t_d}{2}} = \frac{H_b}{\beta_b t_d}, \tag{4b}$$

where: λ_b is the wavelength of the wave in the building, β_b is the shear wave velocity in the building, and t_d is the duration of the half-sine pulse.

The displacement is positive for odd passages and negative for even passages. The displacement and velocity change sign after reflection from the building-half space interface and do not change sign after reflection from the top of the building. The strain changes sign after reflection from the top of the building and does not change sign after reflection from the interface between the half space and the building (Fig. 3a).

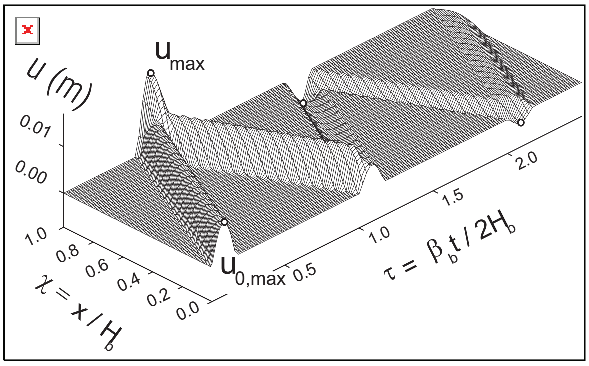

Fig. 3a: Linear displacements along the normalized length of the building, $\chi = x / H_b$, versus normalized time

$\tau = \beta_b t / 2H_b$, for dimensionless pulse amplitude α = 0.03 and dimensionless frequency η = 3

Figs. 3b and 3c illustrate the changes in the nonlinear response, when α = 0.1 and 0.3.

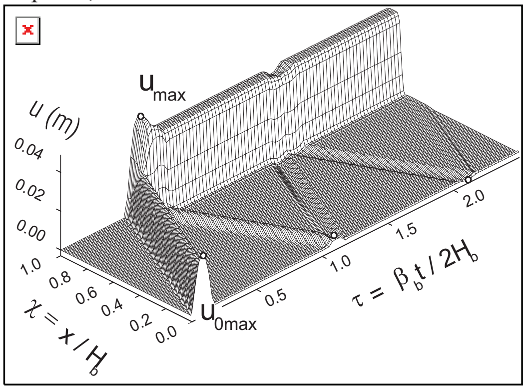

Fig. 3b: Nonlinear displacements along the normalized length of the building, $\chi = x / H_b$, versus normalized time $\tau = \beta_b t / 2H_b$, for dimensionless pulse amplitude α = 0.1 and dimensionless frequency η = 3

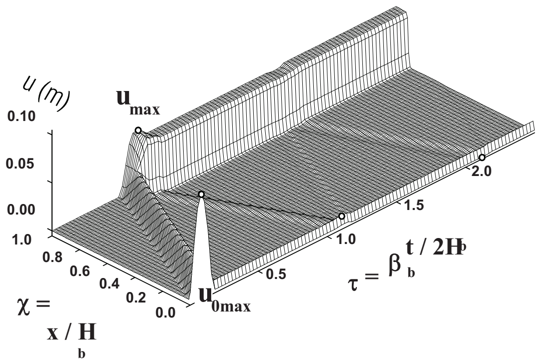

Fig. 3c: Nonlinear displacements along the normalized length of the building, $\chi = x / H_b$, versus normalized time $\tau = \beta_b t / 2H_b$, for dimensionless pulse amplitude α = 0.3 and dimensionless frequency η = 3

Fig.4 shows the difference of the response for the case $\gamma = \frac{\mu_{1b}}{\mu_{0b}} = 0$ (elastoplastic material) and $\gamma = \frac{\mu_{1b}}{\mu_{0b}} = 0.3$ for dimensionless amplitude $\alpha = 0.3$ and dimensionless frequency $\eta = 3$.

It can be noticed that in the case of elastoplastic material (upper plot in Fig.4), there is strain concentration and the zones of large permanent strains are narrow. The zone with largest permanent strain is located near the top of the structure, where due to interference of the reflected from top and incoming from bellow wave, the actual strain exceeds the yielding strain and the material undergoes large permanent strains. Other zone with smaller permanent strain is the interface between the half-space and the layer (building). Notice that for smaller amplitude ($\alpha = 0.1$ in Fig.3b) this zone is not developed.

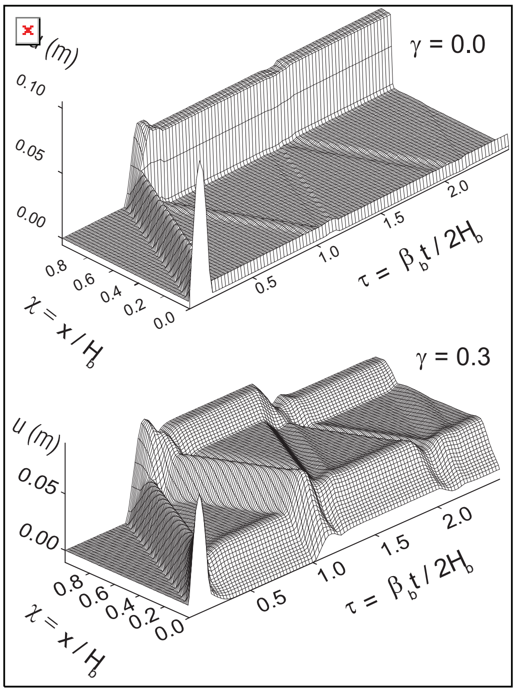

Fig. 4: Plot of the displacements along the normalized length of the building χ, versus normalized time τ, for dimensionless amplitude α = 0.3, η = 3, and for γ = 0, and γ = 0.3

In contrast, for the case $\gamma = 0.3$, the zones of permanent strains are wider, while the value of the permanent strain is smaller than its value for case $\gamma = 0$.

B. Multilayered deposit or multistory building

The model of this case is shown on Fig. 5. This is the model of the North-South response of the Holiday Inn hotel in Van Nuys, California during Northridge earthquake, 1994. This example will illustrate the nonlinear response of a "typical" building supported by a half space with properties that are representative of many metropolitan areas (Fig. 7a). In all calculations in this paper we use H_b = 20.035 m and ε_{yb} = 0.0025 for the maximum linear strain in the bi-linear stess-strain relationship, with second slope γ = 0.44 (Fig. 2), but we present all results in dimensionless terms (4a) and (4b).

The layer stiffness, expressed by shear wave velocity decreases going upward being highest at the base and the lowest at the top story (Fig.5b).

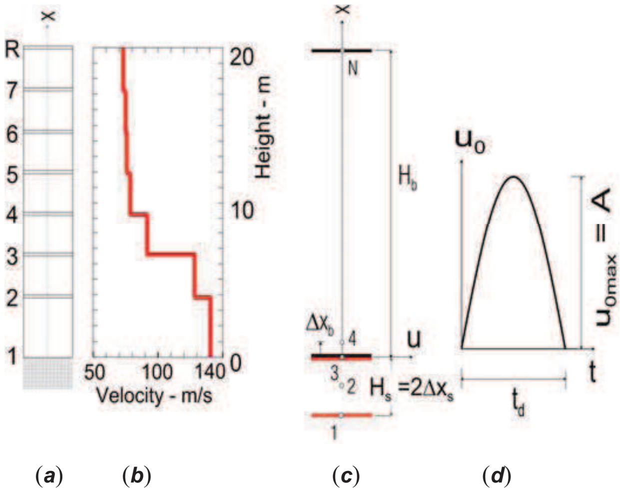

Fig. 5: Building and incoming strong-motion displacement pulse: (a) Layered model of the building, (b) Velocity of shear waves in the layers, (c) Finite difference model with transmitting boundary at node point 1, and (d) the pulse incident from the half space.

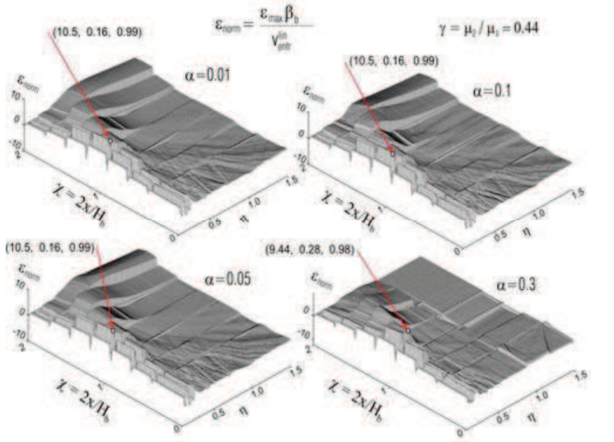

Fig. 6: Normalized peaks of strain $\varepsilon_{norm} = \varepsilon_{\max}\beta_b / v_{entr}^{lin}$, along the normalized building height $\chi = 2x / H_b$, when their maxima occur, versus dimensionless frequency η, $\chi = 2x / H_b$ for $\gamma =$ 0.44, and for four dimensionless amplitudes α = 0.01 0.05, 0.10, and 0.30. Maxima are shown accompanied by their dimensionless coordinates (e.g. (10.5, 0.16, 0.99) for $((\varepsilon_{norm})_{\max}, \eta_{\max}, \chi_{\max})$).

The linear strain is the ratio of the particle velocity and shear wave velocity in the building $\varepsilon_{lin} = \dfrac{v_{entr}^{lin}}{\beta_b}$.

To study the response of this structure, a dimensionless strain is introduced as a ratio of the maximum strain occuring in the beam over the linear strain, $\varepsilon_{norm} = \dfrac{\varepsilon_{\max}}{\varepsilon_{lin}} = \dfrac{\varepsilon_{\max}\beta_b}{v_{entr}^{lin}}$. The results of the response of the building to single pulses with variable dimensionless frequency $0 < \eta \leq 1.5$ and for four values of the dimensionless amplitude α are shown on Fig.6.

Fig. 7a: View of Van Nuys Seven Story Hotel (VN7SH) from North-East

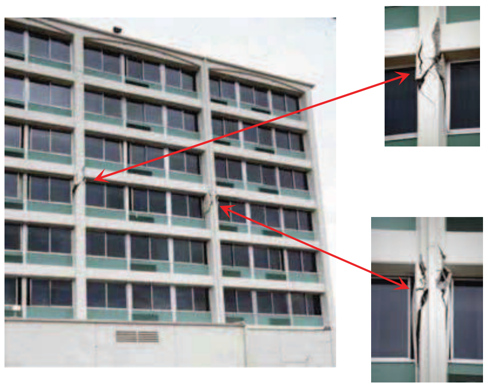

Fig. 7b: Post-earthquake view of damaged columns A7 and A8

Then, the response of the model to the record of strong ground motion during the Northridge earthquake is computed. With comparison of the recorded and computed displacements (Fig. 8), on four different locations in the building, one can notice that there is an excellent agreement between them .

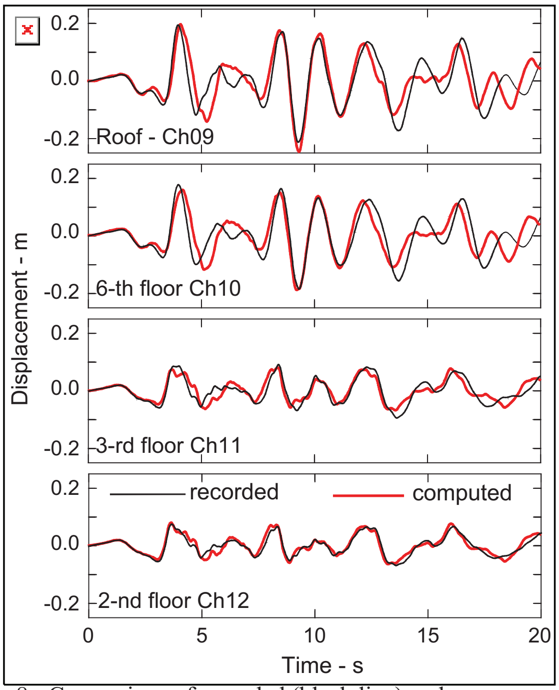

Fig. 8: Comparison of recorded (black line) and computed (red line) displacements.

On Fig. 9, the ratio between actual and yielding strain during time of Northridge earthquake record is presented. It can be noticed that the maximum absolute values exceeding 10 are obtained between ninth and tenth second of the strong-ground-motion record. These values occur between the fourth and the fifth floor, exactly where the maximum damage occurred (Fig. 7b).

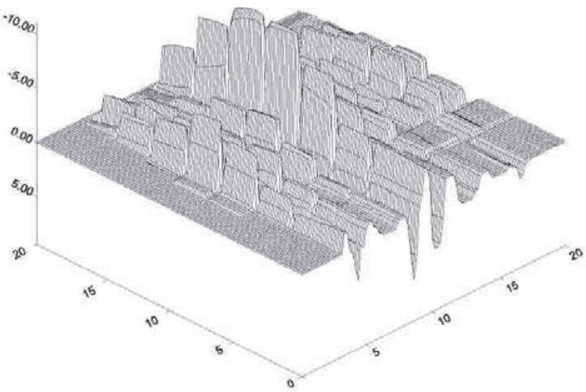

Fig. 9: The ratio $\varepsilon_{perm} / \varepsilon_{yb}$ along the building during the the first 20 sec. of excitation

IV. CONCLUSIONS

From the above analyses the following conclusions can be drawn:

- The scientific computing (computational science) consists of modeling and numerical simulations of physical and engineering phenomena.
- A problem of wave propagation through semi-infinite media was resolved numerically using central finite difference representation.
- The problem was modeled with the simplest, 1-D model of a structure sitting on an elastic soil. The incoming wave was prescribed in the soil and the response of the structure was determined.
- The synthetic displacements (from numerical simulations) were validated with the recorded displacements of the response of Holiday Inn hotel in Van Nuys, California during Northridge earthquake, 1994.
- Even with the simplest, 1-D model, the agreement of the recorded and CFD synthetic displacements is excellent.

V. REFERENCES

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