SIMULATING THE PROFIT OF WORK ON MULTI STATE TWO TERMINAL TRANSPORTATION SYSTEM

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ABSTRACT

In this paper, the work of transportation systems, during the time is examined. We assume that the system works in fixed timed interval. In this time interval, under assumption that the system can't be repaired, we want to compute the total profit of work on system. For that purpose, we make simulation in programming language C#. In the simulation, the exponential distribution is used to simulate the failure time on the components from the transportation system. Also, some examples to illustrate work of simulation are included.

I. INTRODUCTION

Two terminal transportation systems have application in many fields as telecommunication systems, transportation systems, water distribution, gas and oil production and hydropower generation systems [1]. The research of these systems is mainly concentrate on binary approach, but it is found that multi-state theory is more appropriate one. In multi-state theory it is supposed that the system and its components may operate in any of several intermediate states. The literature deals with multi-state systems mostly analyze its reliability, but in this paper, we are concentrating on the profit or the profits that can be obtained during the system operate.

We will regard the system with independent components that works fixed timed interval without reparation. During this time some of its components can fail for some levels and we suppose that on level transitions are homogenous Markov transitions, which imply that one level failure times have exponential distribution. Additionally we suppose that the operations in specifics level for a unit time results with some constant profit. The main idea is to analyze the influence of the profit depending of the structure of the system and the fixed working time without reparation. For that purpose, we found that it is important to simulate such systems in order to obtain visual data that describes this phenomena. In this paper we will explain how we simulate such systems. The simulations are made in programming language C#. In order to be able to make these simulations we need to know minimal path or minimal cut vectors. Some algorithms for obtaining this vectors are proposed in [1] – [4], and we use the algorithm for obtaining minimal path vectors given in [3].

II. SYSTEM DESCRIPTION

Let we have a two-terminal multi state network $G(V, E)$, where V is the set of nodes, and E is a set of links (componets). Let s be the source node and z be the sink node. Such a network is shown in Figure 1.

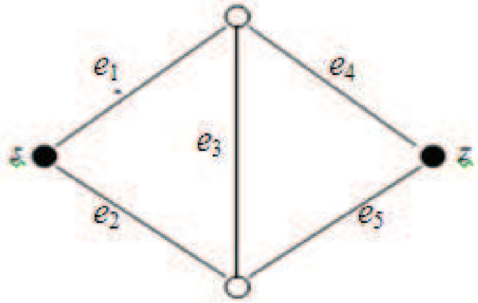

Figure 1

We assume that each of $|E| = n$ components can have capacities 0, 1, …, $M_i + 1$, $i = 1, …, n$ where M_i denotes the maximum capacity that can transit trough the link and 0 is total failure of the components, when there are no flow trough that link. The capacity of the whole system is the capacity that can transit from the source to the sink, and we suppose tha t the available capacities of the whole system are 0, 1, …, M. If u_i is the level of the link e_i, then the state vector is defined as $\boldsymbol{u} = (u_1, u_2, …, u_n)$. For each state vector $\boldsymbol{u}$, the structure function $\varphi(\boldsymbol{u})$ signify the network capacity, i.e. the network flow from source to sink under state u. A vector $\boldsymbol{u}$ is a minimal path vector to level d if $\varphi(\boldsymbol{u}) \geq d$ and for every other $\boldsymbol{u}_1 < \boldsymbol{u}$, $\varphi(\boldsymbol{u}_1) < d$. A vector u is a minimal cut vector to level d if $\varphi(\boldsymbol{u}) < d$ and for every other $\boldsymbol{u}_1 > \boldsymbol{u}$, $\varphi(\boldsymbol{u}_1) \geq d$.

The system runs during the fix time interval T. We define *one state transition time* as the time in which the system transit from state $\boldsymbol{u}$ to state $\boldsymbol{u}$-$\underline{\mathbf{e}_i}$ for some i=1, ..., n and *one level transition* time as the time in which the system fails down for one level, i.e. the capacity of the system is reduced for one level.

By τ_{ij}, i=1, ..., n, j=1,…, M_i, we denote the random variables one level failure times of particular components, i.e. the failure time of the i-th component from state j to state j – 1. According to our assumption that the times have exponential distribution, the appropriate intensities will be denoted by λ_{ij}. If all intensities for one component are equal this intensities can be denoted by λ_i, and the transition times can be denoted by τ_i. Here we regard such type of systems.

One idea for simulation of the system is to simulate all the components separately. Then once we get the one state transition time for all components, we need to find the minimum of these time. For this purpose we sort the one state transition time, by size in sequence. Then we check whether the level of the system will reduce or remain the same. Then, again we simulate the one state transition time for the component that had to minimum time. We put the new time in the sequence of times. Then, we use the same procedure for the component witch time is first in the sorted sequence. If we perform the simulation in this way then we have to compare the one state transition times and that will slow the simulation. In our simulation, we find the join distribution of the one state transition time from all system components.

Since the failure times of the components are independent, τ_1, τ_2, ... , τ_n are independent exponential random variables with parameters λ_1, λ_2, ..., λ_n. If $m \le n$ of this component is in some working states, i.e. there are not state 0, then so the join distribution of this random variables is:

$$\mathrm{P}\{\min\{\tau_i \mid i=\overline{1,m}\} > t\} = \prod_{i=1}^{m} P\{\tau_i > t\} = \prod_{i=1}^{m} e^{-\lambda_i t} = e^{-\sum_{i=1}^{m}\lambda_i t} \quad . \qquad (1)$$

This means that one state transition time of the system has exponential distribution with parameter $\lambda_1 + \lambda_2 + ... + \lambda_m$.

This is used in our simulation on the way that we generate data sequence with such distributions. In the beginning we may suppose that the system starts in its perfect state, and in this time the parameter is $\lambda_1 + \lambda_2 + ... + \lambda_n$. But, during the time T some of the components can take the level 0, so the parameter become $\sum_{i,u_i>0} \lambda_i$. In each step, one of the components fails down for one level, but (2) do not tells us which component is that one. So we need the further analyze. The probability that one state transition become as a result of failure of the i-th component is

$$P\{\tau_i = \min_{r,ur>0} \tau_k\} = \frac{\lambda_i}{\sum_{r=1}^{m}\lambda_r}. \qquad (2)$$

If $\lambda_1 = \lambda_2 = ... = \lambda_n = \lambda$ we obtained:

$$P\{\tau_i = \min_{r,ur>0} \tau_r\} = \frac{\lambda}{n\lambda} = \frac{1}{n}. \qquad (3)$$

III. Profit of the System

Suppose that during the time interval T, the system make k one level transitions, that may be in times t_1, t_2, t_3, ... t_k, where $T < t_1 + t_2 + t_3 + ... t_k$, Figure 2.

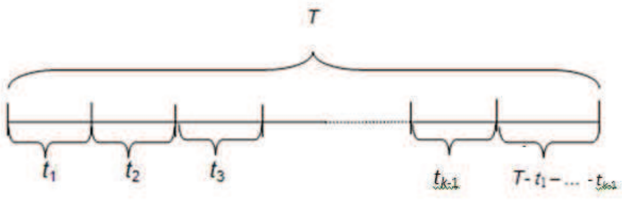

Figure 2

The profit per unit time when the system works in state d without failure will be denoted by C_d. Initialy, the flow from source to sink is M. The capacity of the system can be reduce by 1 only when some of the components fail, but in some cases, after one state transition, the capacity can remain the same. Let in the moment t_1, the flow is reduced by 1, i.e. the system works in level M time t_1. The profit in this time interval with length t_1 is $C_M t_1$. Then, if the system works in level $M - 1$ time t_2, then the profit for this is $C_{M-1}t_2$. Continuing on that way, for the time t_i profit is $C_{M-i+1}t_i$. The same procedure is repeated, until $T < t_1 + t_2 + t_3 + ... t_k$ for some k. The profit for the last interval is

$$C_{M-k+1}(T - (t_1 + ... t_{k-1})).$$

Now, the total profit during time T is

$$C_{total} = C_M t_1 + ... + C_{M-k+2} t_{k-1} + C_{M-k+1}\,(T - (t_1 + t_2 + ... t_{k-1})) \qquad (4)$$

Let, w_i $i=1...r$ are one state transition times. System capacity can be reduced only when a component falls. If capacity of the system is reduced, then the one state transition times matches to the one level transition times, Figure 3

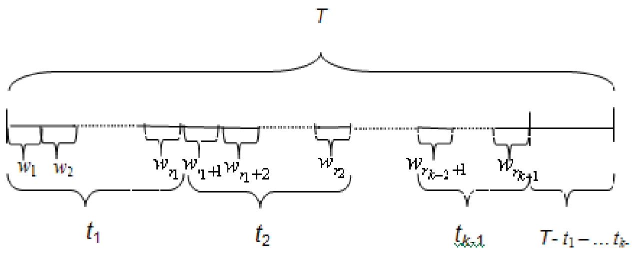

Figure 3

Since

$$t_1 = w_1 + w_2 + \ldots + w_{r_1}$$
$$t_2 = w_{r_1+1} + w_{r_1+2} + \ldots + w_{r_2}$$
$$t_{k-1} = w_{r_{k-2}+1} + w_{r_{k-2}+2} + \ldots + w_{r_{k-1}}$$
$$t_k = w_{r_{k-1}+1} + w_{r_{k-1}+2} + \ldots + w_{r_k} \qquad (5)$$

We can use the expressions t_i $i=1,2,\ldots k$ in (4) and we get the following equation

$$C_{total} = C_M(w_1 + w_2 + \ldots + w_{r_1}) + \ldots$$
$$+ C_{M-k+2}(w_{r_{k-2}+1} + w_{r_{k-2}+2} + \ldots + w_{r_{k-1}})$$
$$+ C_{M-k+2}[T - ((w_1 + w_2 + \ldots + w_{r_1})$$
$$+ (w_{r_1+1} + w_{r_1+2} + \ldots + w_{r_2}) + \ldots$$
$$+ (w_{r_{k-2}+1} + w_{r_{k-2}+2} + \ldots + w_{r_{k-1}})] \quad (6)$$

IV. DESCRIPTION OF THE SIMULATION

In this part, we will explain how the simulation works. For a given fix time interval T, network with links $e_1, e_2, \ldots e_n$, with maximal capacities $M_1, M_2, \ldots M_n$ and failure intensities $\lambda_1, \lambda_2, \ldots, \lambda_n$ we have separate procedure that calculate the system capacity $\varphi(\boldsymbol{u})$, when the system is in state $\boldsymbol{u}=(u_1, u_2, \ldots u_n)$. For this part we use the algorithm for minimal path vectors given in [3].

In each step, when the system works with level d, $0 < d \leq M$, and the calculated profit to that time is C_{total}, we simulate exponential distribution with parameter $\sum_{i,u_i>0} \lambda_i$. In the first step this parameter is equal to $\lambda_1 + \lambda_2 + \ldots + \lambda_n$. In the r-th step we obtaine the time w_r. The total profit of the system will be calculate with the following equation:

$$C_{total} = C_{total} + C_d w_r.$$

The selection of the fail component, during time w_k is doing using the procedure **ran1**($\lambda_1, \lambda_2, \ldots, \lambda_l$) on the following way. If the set of the components that are in some working level is $S_r = \{u_{r_1}, u_{r_2}, \ldots, u_{r_l}\}$, we choose a random number a from the interval $[0, \sum_{u_i \in S_r} \lambda_i]$. We decide that the fail component is the i-th component from S_r if

$$\lambda_{r_1} + \lambda_{r_2} + \ldots + \lambda_{r_i} < a \leq \lambda_{r_1} + \lambda_{r_2} + \ldots + \lambda_{r_{i+1}}.$$

Then we construct new system from the old one by reducing the appropriate component for one level down. After that, by the procedure **cheking_level** we check whether this new system reduces the capacity. This is done by checking does the new state vector $\boldsymbol{u}$-$\underline{\mathbf{e}_i}$ is greater than some of the minimal path vectors for level $d = \varphi(\boldsymbol{u})$. If it is a case, than the flow has not be reduced, but if new state of system is not greater than any minimal path vector, flow will be reduced and it becomes $d - 1$.

```
Input: u = (u1, u2, ... un), minimal path
vectors Vd ={v1, v2, ... , vl } for level d)
Output: (boolean variable reduce)
Procedure cheking_level(u, Vd)
bool  reduce = true
For i =1 to l
    if ( u > vi )
       reduce = false
```

The simulation works as long as the sum of the exponential times $w_1 + w_2 + \ldots + w_l$ is less than T or if there not flow from the source node to the sink node.
The pseudocode is:

```
Input: weighed graph G(V, E), set of profits C={C1, ..., CM}, set of intensities Λ= {λ1, λ2, ..., λn } Output:
total profit

Program calculate_profit(G(V, E), C, Λ)
  Call the procedure minimal_path_vector for
  obtaining all minimal path vectors for all
  level.
  w = 0;
  Ctotal =0;
  d=M;
 while  (w < T or M > 0);
 {Select random number ran;
  wr = -((Σ_{i,ui>0} λi) log(1 - ran))^(-1) ;
  if (w> T) wr = wr - (w - T);
  w = w + wr;
  Ctotal = Ctotal +  Cd wr.
  ran1(Λ);
  If   cheking_level(u,v1,... ,vl )=true
       {d = d -1;
        u = (u1, u2, ... uran1 - 1,...un);
       If(uran1=0)Λ={λ1,λ2, ...,λn}-{λran1}}
}
```

V. CASE STUDY

In this part we will explain the proposed simulation by an example. We consider the network given in Figure 1. All intensities parameters a set on $\lambda = 2$ and the profit for level i is set on $2i$. The maximal state vector of the system is (2,1,2,3,2). It is clear that the maximal capacity level is 3. The minimal path vectors for levels 1, 2 and 3 are given in Table 1.

d	Minimal path vectors for level d				
	e_1	e_2	e_3	e_4	e_5
1	1	0	1	0	1
	1	0	0	1	0
	0	1	1	1	0
	0	1	0	0	1
2	2	0	2	0	2
	2	0	1	1	1
	1	1	1	0	2
	2	0	0	2	0
	1	1	1	2	0
	1	1	0	1	1
3	2	1	1	1	2
	2	1	1	3	0
	2	1	0	2	1

Table 1. Minimal path vectors for level 1, 2 and 3 for the network from Figure 1.

w_r	e_1	e_2	e_3	e_4	e_5	d	u	*Profit*
1.3557	**2**	1	2	3	2	3	u_1	8.1345
0.0916	1	1	2	3	**2**	2	u_2	0.3665
0.3397	1	1	**2**	3	1	2	u_3	1.3589
0.2346	1	1	1	**3**	1	2	u_4	0.9385
1.3911	**1**	1	1	2	1	2	u_5	5.5644
1.2077	0	1	1	**2**	1	1	u_6	2.4155
0.4376	0	1	**1**	1	1	1	u_7	0.8752
0.5677	0	1	0	1	**1**	1	u_8	1.1355
2.3412	0	1	0	1	0	0	u_9	0
total profit								20.7891

Table 2. Profit for network given in Figure 1 with initial state vector (2,12,3,2) and $T = 20$

The steps of simulation for fix time T=20 are given in Table 2. The failure time w_i are given in the first column, the second column correspond to state vectors obtained after each one state transition vectors. The bold numbers represent the component which failed in the moment w_r. The new flow d is given in the 3-th column, and in the fourth one is given the particular profits obtained in appropriate steps. It is obvious that for first time the flow is reduced in step 2. The total profit is 20.7891. In this case, the simulation stops before time T, since the system came in the level of total failure.

This simulation runs 100 times and the obtained distribution of the total profit is shown in Figure 3.

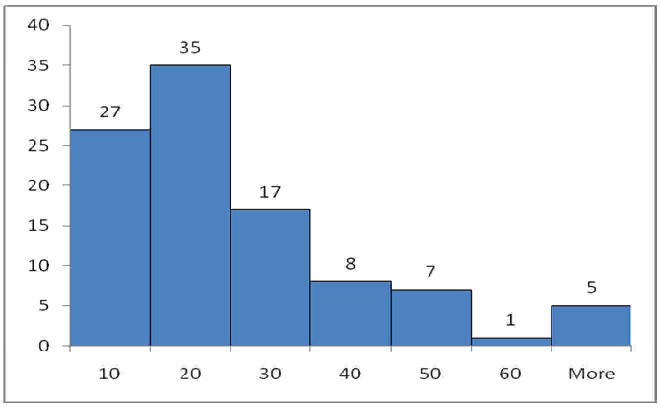

Figure 4

The next experiment we show is for the same system, but the fix time interval during we consider the simulation is $T = 2$. The result of one experiment is given in Table 3.

w_i	e_1	e_2	e_3	e_4	e_5	d	u	*Profit*
0.199	**2**	1	2	3	2	3	u_1	1.1942
0.2002	1	1	2	3	**2**	2	u_2	0.8010
0.8211	1	**1**	2	3	1	2	u_3	3.2847
0.0845	1	0	2	3	**1**	1	u_4	0.1691
0.6952	1	0	2	3	0	1	u_5	1.3898
total profit								6.84

Table 3. Profit for network given in Figure 1 with initial state vector (2,12,3,2) and $T = 2$

This simulation finished in the fifth step, since in the next step time $w_5 = 4.2353$ and $w_1 + w_2 + w_3 + w_4 + w_5 > T$. In calculation of the total profit instead w_5 we take 2 – (0.199+0.2002+0.8211+0.0845) = 0.6952.

This simulation also runs 100 times and the obtained distribution of the total profit is shown in Figure 4.

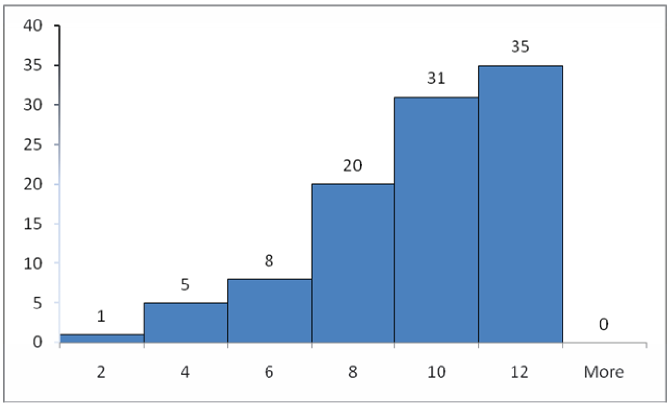

Figure 5.

We can observe that the distribution of the total profit do not have some famous distribution and the distribution depends from the chosen value for T.

Next, for 100 data obtained from all simulations we find the average value. Then we run the simulation once again for 100 times and one more time we find the average value from obtained data. By repeating the procedure 100 times we get a 100 average values for the total profit. Distribution of the average values from the total profit when $T = 20$ is given in Figure 6 and for $T = 2$ in Figure 7.

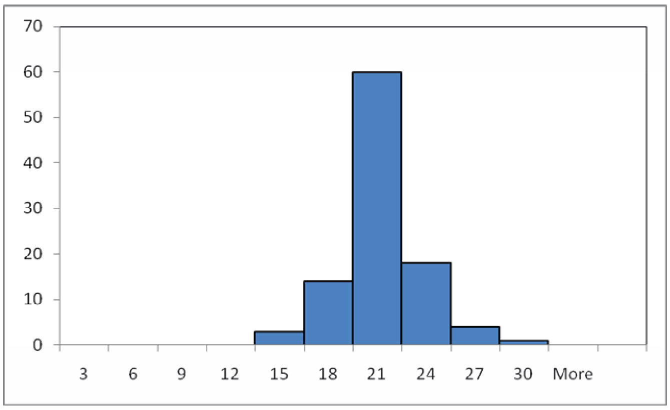

Figure 6

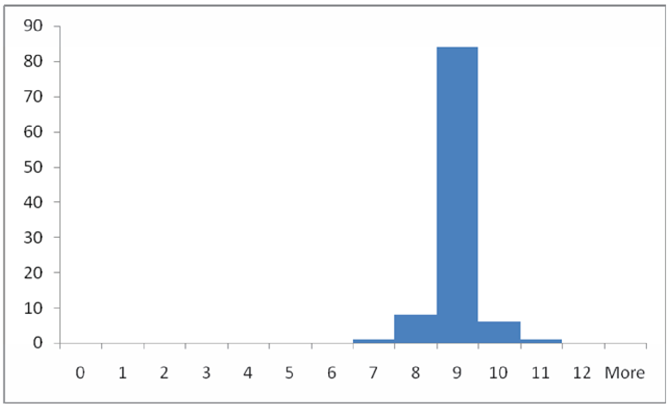

Figure 7

We can observe that the distribution of the average of total profit have normal distribution. In the case, when $T = 20$, most data are between 24 and 27, and when $T = 2$ between 9 and 10.

From the other side, although the fix time T in the first experiment is 10 times bigger then the fix time T in the second experiment, the mean value is around 2 times bigger. This phenomenon occurs because a great number of systems in the first experiment a found in the state of total failure, around 90%, before time T.

Usually the time T is chosen as a time in which we will decide weather the system be repaired. So it is not useful to take a time T as in the first experiment because with such a choice the probability that the system will not work for a large part of T is big one. One interesting question is to find an optimal value for this time in respect to maximize the total profit if in this time we may get decision does the system is useful to be repaired or not.

VI. Conclusion and Future Work

In this paper we present simulation for calculating a total profit on two terminal multi state transportation system. Some experimental results are shown. Further research will be directed to finding a mathematical model for distribution the total profit. The simulation will be used for comparing theoretical results and their confirmation. Our next research will be focused on optimization algorithms of the profit of such systems and these simulations will be useful tool for that analyze.

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