

## COMPASS AND STRAIGHTEDGE WITH GEOGEBRA USING TECHNOLOGY TO MASTER MATHEMATICS

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### ABSTRACT

In this paper, we show how to use the free and open source software (FOSS) GeoGebra to do compass and straightedge (C+S) constructions in exactly the same way as using the actual tools. In this way the skills are completely transferrable in both directions. Because GeoGebra is FOSS, both teachers and pupils can freely practice and learn these techniques. We show the 2 pre-prepared GeoGebra worksheets that come with the basic and advanced C+S toolbars and demonstrate their use. Finally, our team has translated all these materials (as well as GeoGebra and the help and the introduction books) into Macedonian, so C+S with GeoGebra is available totally in the language of our pupils and students.

KEYWORDS: Compass, Straightedge, Ruler, GeoGebra

### I. INTRODUCTION

Let us briefly discuss the benefits of doing compass and straightedge constructions and why we would want to do this with a dynamic math program (and not just with a real compass and straightedge) and then why we think GeoGebra is an excellent choice for this.

#### 1) Why do we study compass + straightedge constructions?

- It is fun and allows creativity.
- It teaches logical thinking skills.
- It is required learning

2) Why do we want to do compass + straightedge with a dynamic math program? The answer came from necessity. In trying to both understand the why and how of an advanced C+S construction by hand, it became obvious that a dynamic math program would help us both explore the construction to see the vital points and allow us to try different approaches. Further, a dynamic math program allows panning and zooming so the construction is always the "right size" and in the "right place" so we don't have to keep starting over and over or erasing.

#### 3) Why GeoGebra?

GeoGebra is an excellent dynamic mathematics program. It is installed on every computer in the "a computer for every pupil" program in R. Macedonia. Here are some of the reasons GeoGebra was a good choice.

- FOSS (Free and open source software)
- Can be used on school LANs
- Can be used offline
- The program is translated into many, many languages including Macedonian

- In addition to C+S constructions, GeoGebra has many, many applications in algebra, geometry, calculus, statistics, etc.
  - Ready to use applets are available
  - Is easy to use, powerful and dynamic
  - Teachers and students can work anywhere
  - Students can build animations, simulators, ...
  - Has algebra, geometry and spreadsheet

#### 4) Why use GeoGebra for C+S constructions?

It is said that the most effective educational technologies are the ones that require little or no behavioral change on the part of the teacher or the learner. They should not increase cognitive load, nor should they require extensive training or technical support. (Dr. Andrew Van Schaack, Vanderbilt U.)

GeoGebra is the only dynamic mathematics program with all of the tools necessary for C+S constructions and a way to limit the toolbar to these tools. The special compass tool was created LFS to imitate a compass. This tool was then submitted and incorporated into GeoGebra 3.2 and is now a standard tool. We discuss the C+S toolbar in detail below, but it is important that the steps in a by-hand construction are exactly the same as in a GeoGebra construction. This allows the teachers and pupils an enormous amount of flexibility.

Also GeoGebra is very easy to use to explore [6]. While it is important to do C+S constructions, it is even more important that pupils understand how to reason through a construction and not just learn the steps by memory. We discuss this process in detail in II. Logical Thinking Skills and in V. Advanced C+S Constructions with GeoGebra

By using GeoGebra, the pupil can understand both the why and the how of C+S constructions.

### II. LOGICAL THINKING SKILLS

One of our main goals in mathematics education is teaching logical thinking skills. That is, the ability to look at a problem and see what is given, what is required and the steps needed to get from the given to the required and how to check the final result. How do compass & straightedge (C+S) constructions help us learn logical thinking skills?

With C+S constructions:

- We have very, very small set of tools.
- The problems state very specifically what is given and what is required.
- Every construction is based on previous constructions, that is, on previous step-by-step learning.

- The construction is visible and measurable. We can see it and we can check it physically to see whether it is correct.

An added bonus of doing C+S constructions with GeoGebra is that this process is very clear and the ability to check a construction with GeoGebra is both easy and instructional.

### III. USING GEOGEBRA FOR C+S

#### A. C+S in GeoGebra

Here we discuss the basics of using GeoGebra for C+S constructions.

##### 1) *BasicConstruct.ggb*

Ready-to-use worksheet with the GeoGebra toolbar for C&S Available online (post it freely) at:

<http://mathcasts.org/gg/construct/BasicConstruct.html>

Download and open it with GeoGebra

<http://mathcasts.org/gg/construct/BasicConstruct.ggb>

##### 2) *Construction Protocol*

#### B. C+S Toolbar in GeoGebra

GeoGebra has many, many tools and the ability to create complex objects with a single command. However our goal here is to learn C+S constructions using exactly the same skill set as when doing physical C+S so that the skills are transferable in both directions.

What tools do we have with physical C+S? We have a pencil, a compass and a straightedge. What are these tools in GeoGebra?



Figure 1: Main tools in the C+S Toolbar in GeoGebra

##### 1) *Pencil tools*

With a pencil, we draw new points in the drawing space and also mark intersection points. For these actions, we use the new point tool and the intersection tool in GeoGebra. New points are free objects and can be moved. Intersection points are dependent on the objects of intersection and cannot be moved on their own. However, they will move automatically if their defining objects are moved.

##### 2) *Straightedge tools*

With a straightedge we draw lines, line segments and rays. We use the corresponding tools in GeoGebra. However, we note that GeoGebra will also draw the two points on which each of these objects depend. The points are free objects, whereas the line, segment and ray are dependent objects. We move the dependent objects by moving the free objects on which they depend.

##### 3) *Compass tool*

GeoGebra 3.0 did not have a compass tool. So the authors created one and it is now integrated into GeoGebra 3.2.

With a compass, we measure a length between 2 points for a radius and then draw a circle with center at a 3rd point with that radius. With the compass tool in GeoGebra, we click successively on the 2 endpoints of a length (line segment) to get the radius and then drag the circle to the center point and click. The circle is a dependent object. Its radius and center will change if the corresponding points are moved.

##### 4) *Other tools*

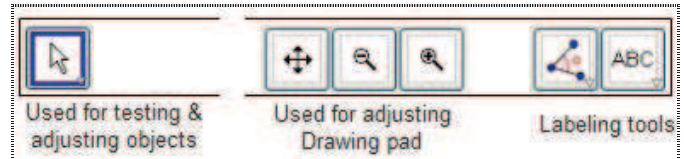







Figure 2: Other Tools in the C+S Toolbar in GeoGebra

The other tools in the C+S toolbar in GeoGebra are “helping” tools. The *Move tool*  is used to move free objects. The *Move drawing pad*  is used to shift (pan) the drawing pad. One can also move the drawing pad by pressing the Shift key and then positioning the mouse pointer in an empty space and using click + drag.. Zooming can be done with the mouse scroll button.

Finally, we have the “writing with a pencil tools”.

The *Angle through 3 points tool*  is **used here only as a labelling tool**. It is used to “draw” the arc of an angle created with the straightedge tools.

The *Insert text tool*  and (below it) the *Distance tool*  are also used only as labelling tools.

### IV. RESOURCES TO LEARN

We developed online resources to help teachers and pupils learn to use GeoGebra and learn to do C+S constructions with GeoGebra. These resources are available on [1] in English and [2] in Macedonian. There is a page on C+S with GeoGebra and the file *BasicConstruct.ggb* and then there are 7 pages with the 7 basic C+S constructions. Each page contains

- YouTube Mathcasts with audio and visual explanations of doing the construction.
- Online interActivity with the construction already done in GeoGebra for exploring and testing.
- Printable and online materials with explanations directions for classroom and educator.
- Online DIY blank GeoGebra worksheet for doing the construction online.
- Metadata about the resource.
- Links for downloads of all materials for offline and school LAN use.

#### A. *Links*

The main link for the C+S materials in English is:

<http://geogebrawiki.pbworks.com/Compass-and-Straightedge>

The main link for the C+S materials in Macedonian is:

<http://geogebrawiki.pbworks.com/Шестар-и-линијап>

### B. Seven Basic Constructions

In order that the processes are completely clear, we detail the seven basic constructions with GeoGebra.

- #1 Copy a line segment
- #2 Copy a triangle
- #3 Copy an angle
- #4 Bisect a line segment
- #5 Draw a normal
- #6 Bisect an angle
- #7 Draw a parallel

In this way both pupils and teachers have complete audio-video lessons and worksheets of all the standard actions so that there is no confusion.

### V. ADVANCED C+S

In the classroom, with advanced C+S constructions, pupils are allowed to use 2 similar triangles in order to quickly draw normal (perpendicular) and parallel lines. It is understood that they know how to do this with just a compass and a straightedge (#4 and #7), but in the interest of saving time they are allowed to skip this part and just use the triangles.

#### A. *AdvancedConstruct.ggb*

These tools are included in the advanced C+S worksheet in GeoGebra: *AdvancedConstruct.ggb* (see: Figure 3).

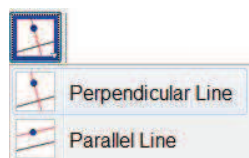


Figure 3: Perpendicular and Parallel tools

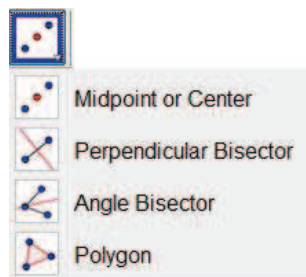


Figure 4: Other tools

In addition, if the teacher wishes to allow it and for the same reasons, the tools for drawing a segment midpoint (#4), a normal bisector (#5), an angle bisector (#6) and a polygon are also given on this GeoGebra worksheet (see Figure 4).

#### B. *Inscribe a semi-circle in a square*<sup>1</sup>

We choose this problem to illustrate the “*explore+ construct*” aspects of GeoGebra.

##### 1) Step 1: Understand the problem

From the definition of “inscribed”, it follows that the two endpoints of the diameter of the semi-circle must lie on the

<sup>1</sup> Inspired by Colin McAllister [8]

edges of the square and that we are looking for the largest such semi-circle that lies within the square. We start by exploring whether these two points lie on the same, on opposite or on adjacent sides of the square.

##### 2) Step 2: Exploring

Construct a dynamic square with side  $a$ . We create two dynamic points  $E$  and  $F$  on the edges of the square and a semi-circle with diameter  $d=EF$  as follows:

- $E$  and  $F$  **on base of square** (see Figure 5)<sup>2</sup>. What is largest inscribed semi-circle? Clearly if  $E$  and  $F$  are stretched to the base vertices so  $d=a$ .

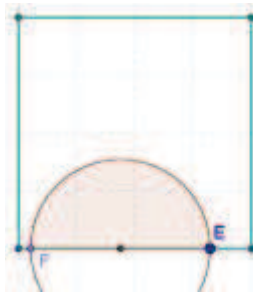


Figure 5:  $E$  and  $F$  on same side

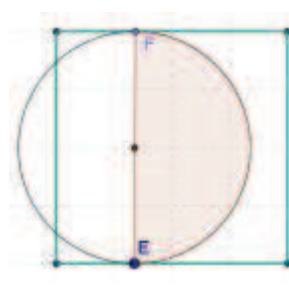


Figure 6:  $E$  and  $F$  on opposite sides

- $E$  and  $F$  **on opposite sides of square**. Circle is inscribed only if  $EF$  is parallel to side of square and again  $d=a$  (see Figure 6).
- $E$  and  $F$  on adjacent sides of square. These are larger semi-circles (see Figure 7) Largest inscribed semi-circle is with both “sides” touching (see Figure 8).



Figure 7:  $E$  and  $F$  on adjacent sides; one side touching.

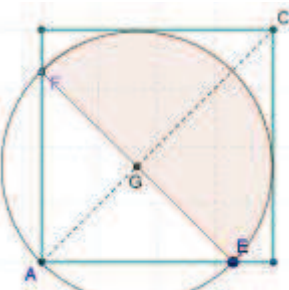


Figure 8:  $E$  and  $F$  on adjacent sides; largest inscribed semi-circle

##### 3) Step 3: Properties of Good Circle

What are the properties of this largest inscribed semi-circle? Things we can easily notice: #1  $E$  and  $F$  are equidistant from  $A$ , #2 segment  $EF$  is normal to diagonal  $AC$ , #3 the center of circle  $G$  is on diagonal  $AC$ , #4  $|AG|=|EG|=|FG|$  are all radii of semi-circle.

**Side note:** By experimenting – that is, by moving any dynamic point – we can see that regardless of the relative positions of  $E$  and  $F$  on adjacent sides, the circle with diameter  $EF$  ALWAYS goes through  $A$ . We can then have a discussion

<sup>2</sup> We draw a semi-circular sector to make the exploration clearer.

about this and conclude the inverse of Thales' theorem. This can be a whole separate exploration.

Make a new drawing with dynamic square, diagonal AC, dynamic point G on AC, normal to AC through G, intersection points E and F.

4) Step 4: Find Construction Key

Draw normals to base and side of square through G. Notice that intersection points of these normals with sides coincide with intersection points of semi-circle and sides. Label them H and I (see Figure 9).

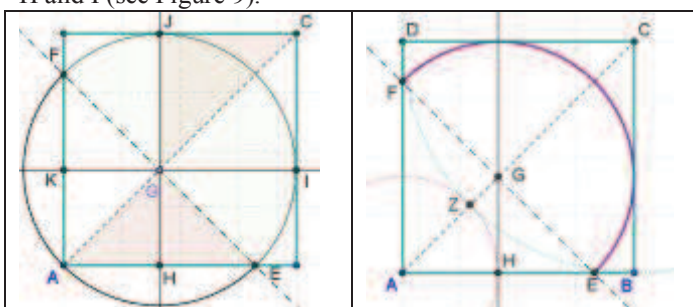


Figure 9:  $AGE \cong GJC$

Figure 10: Final Construction

With our drawing we can see that:

- To do the construction, we need a length for AH or AG or AE. (Any one of these determines the semi-circle.)

The important thing to remember is that we can measure only the side of the square  $a$  and the diagonal AC.

- We can see that  $\triangle AGE$  is congruent to  $\triangle GJC$ .
  - So  $|AG|=|JC|$  and  $|GC|=|AE|=|2AH|$ .
- Easy to see that  $|AH|+|JC|=|a|$  (1)
- Easy to see that  $|AG|+|GC|=|AC|$ 
  - So  $2|AH|+|JC|=|AC|$  (2)

Subtracting (1) from (2), we have  $|AH|=|AC|-|a|$ . So we have a length for AH in terms of AC and a!

5) Step 5: Do Construction

We give the construction for completeness. Of course, this can be done many different ways and pupils should be encouraged to be creative. Draw square ABCD and the diagonal AC. Draw circle with radius  $a=|AB|$  at C. Find the intersection point Z along the diagonal AC.  $|AZ|$  is the length of AH. Draw circle with radius  $|AZ|$  at A and mark the intersection point H along AB. Draw a normal to the base through H and mark the intersection point of G of this normal and AC. Draw a normal to AC through G. Mark the intersection point E and F of this line and the square. Draw circle with radius GE at G. (If you have all GeoGebra tools, draw a Semi-circular arc with center G between 2 points E and F.) See <http://geogebrawiki.pbworks.com/SemiCircle+in+Square>

VI. C+S WITH GEOGEBRA IN R.MACEDONIA

In R. Macedonia, basic and advanced C+S constructions are part of the required curriculum. At both levels many of the constructions are quite difficult to understand and motivate

without exploration software. The tendency is to simply memorize the steps in a few constructions and hope that these will be the ones asked on the test. Of course, this totally defeats the purpose of learning C+S constructions.

On the other hand since 2008, GeoGebra has been pre-installed on all the computers - including the computers for the teachers - as part of the "a computer for every pupil/teacher" programs. The program GeoGebra, the GeoGebra Help book and the Introduction to GeoGebra book have all been translated into Macedonian by the authors, headed by Linda Fahlberg-Stojanovska and our team at TFB in Bitola.

Further, the authors have created a wiki – web portal <http://geogebrawiki.pbworks.com>. This is also the official website of the GeoGebra Institute of R.Macedonia (IGMK). Also all of the above mentioned resources for C+S constructions as well as many other GeoGebra related mathematical resources are available in Macedonian and every day this body of material grows.

So all of the conditions for the successful integration of GeoGebra into C+S in R. Macedonia are in place.

VII. SUMMARY

Compass and straightedge constructions are a fun, useful, and usually required part of elementary and middle school curricula. With a very minimum of training one can easily integrate the FOSS GeoGebra into the daily curriculum as both an exploratory and construction tool. This will help teachers and pupils to motivate and to increase understanding of these constructions, which play an essential role in teaching logical thinking skills that is one of the primary goals of mathematics education.

VIII. REFERENCES

[1] <http://geogebrawiki.pbworks.com> (GeoGebra)  
 [2] <http://geogebrawiki.pbworks.com> (GeoGebraMK, IGMK)  
 [3] <http://www.mathopenref.com/> (compass + straightedge)  
 [4] GeoGebra YouTube Channel  
<http://www.youtube.com/geogebrawiki>  
 [5] <http://math247.pbworks.com/> (mathcasts, pencasts, ...)  
 [6] Fahlberg-Stojanovska L., Stojanovski V. (2009) *GeoGebra: Freedom to Explore and Learn. Teaching Mathematics and its Applications*. Oxford Journals, Oxford, UK, vol.26, no.1, pp.17-22. [teamat.oxfordjournals.org/cgi/content/abstract/hrp003](http://teamat.oxfordjournals.org/cgi/content/abstract/hrp003)  
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