ON STOCHASTIC MODELLING AND PERFORMANCE ANALYSIS OF E-CUSTOMER'S ONLINE BEHAVIOUR

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ABSTRACT

The contemporary ways of doing businesses within the global economic environment, relaying massively on e-Commerce and e-Business paradigms, have resulted in the appearance of a brand new type of virtual customers, new Web-based trading and shopping tools and media, as well as a plethora of novel human skills, knowledge, behaviour, and interactions. As volume of online transactions increases significantly on a daily basis, resulting in higher workloads, the question of assuring pertinent levels of Quality of Service (QoS) becomes an imperative for the Web sites running e-Commerce applications. The continually increasing demands for building and maintaining suitable IT infrastructure have led to the necessity of building adequate predictive models in order future load levels to be foreseen. Within the paper the focus has been put on modelling e-Customer's online behaviour, as being a promising basis for carrying out performance analysis and evaluation needed for the capacity planning process. In addition, it will be shown that an e-Customer's online interaction can be seen as a stochastic process, whose modelling and solving approach anticipates the use of Stochastic Petri Nets (SPNs). The paper also highlights a number of key elements regarding the employment of the class of Deterministic and Stochastic Petri Nets (DSPNs) for modelling the stochastic behaviour, i.e. the interaction between e-Customer and e-Commerce Web site, based upon the use of a corresponding Customer Behaviour Modelling Graph (CBMG). The framework being proposed represents a solid basis for obtaining relevant performance indicators, still remaining a platform for further research in this area.

I. INTRODUCTION

The concepts of e-Commerce and e-Business have indeed matured since the years of 1998-2000, when they were considered euphoric and exotic technologies, rather than usual and useful ones. These paradigms now represent the foundation of most businesses worldwide. Banking, travel, entertainment, shopping and e-mailing now are all part of a new, online, real-time mode in which the Internet society interacts. Since then, markets in western countries of America and Europe have warmed up to online shopping in a big way and now online transactions form a significant part of the total trade in these countries. Several factors have contributed to this phenomenon. Greater Internet penetration, fall in prices of hardware, fall in the price of Internet communication, high bandwidth becoming readily available, development of better and more reliable Web technologies, and increased awareness among users are few of the prominent factors leading the big shift from the traditional, "brick-and-mortar", towards the novel, "click-and-mortar" business model.

As the intensity of online transactions increases considerably day-by-day, resulting in ever higher workloads, it puts additional stress on Web site's hardware resources. The question of assuring relevant Quality of Service (QoS) levels becomes an imperative for any e-Commerce Web site, which is equivalent to assuring e-Customer's satisfaction and high expectations while using e-Commerce environment for virtual buying, selling or trading. This aim can be achieved through the process of capacity planning, which is a crucial part of every e-Commerce Web site deployment.

According to Menascé & Almeida [1, 2], "... capacity planning is the process of predicting when the future load levels will saturate the system and determining the most cost-effective way of delaying system saturation as much as possible", based on natural evolution of the existing workload, the deployment of new Web applications and services, and taking into account the unpredictable and stochastic changes in e-Customer's online behaviour. They have shown, through a pragmatic approach, that it is possible to build up predictive performance models for e-Commerce infrastructure that can be used for capacity planning, based on the usage of queuing networks and basic performance laws, including the Utilization Law, Forced Flow Law, Service Demand Law and Little's Law. Building such predictive models in a methodical and quantitative way is 'a must' for e-Commerce Web sites in order to avoid losing e-Customers due to site crashes or poor performance, as well as to evaluate and plan the Web site to avoid frequent upgrades and migrations. Their approach relies on building predictive models, made at hardware resource level, which, in turn, are based on the corresponding workload model. The later one, in turn, can be derived from the related e-Customer behaviour model.

II. MODELLING E-CUSTOMER'S ONLINE BEHAVIOUR

Customers of a particular e-Commerce Web site interact with it through a series of successive and mutually correlated requests, being made during a single shopping session. Within an online session, e-Customers can issue requests of different types, such as Login, Browse, Search, Add to Cart, or Pay. Moreover, different e-Customers may reveal different patterns of navigation through an e-Commerce Web site, invoking the different specific functions, provided by the Web site, in different manner (sequence) and with different frequencies. By invoking a single function with a single click, an e-Customer actually contributes to the overall Web site's workload, by putting a substantial amount of service demand on its hardware resources (CPU, I/O subsystem, HDD, network, RAM etc.). Thus, it is important, if not crucial, to be able to characterize the e-Customer's online behaviour. It is a first step towards building a corresponding workload model, which, in turn, is an input for building a relevant resource model, necessary for carrying out a performance analysis [1].

The Customer Behaviour Model Graph (CBMG), originally introduced by Menascé & Almeida (2000) [1, 2], identifies and represents the available e-Commerce specific functions of a particular e-Commerce Web site, as well as the implemented business logic in the most appropriate and visual manner (Fig. 1). It captures the key elements of e-Customer's online behaviour in terms of navigational patterns, e-Commerce functions being used, frequency of access to various e-Commerce functions, and times between accesses to the various services offered by the Web site [1, 2].

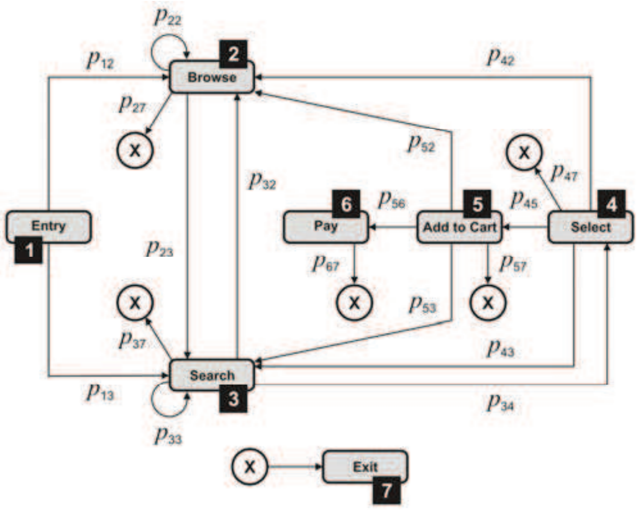

Figure 1: A CBMG, depicting an e-Customer's online interaction with a particular e-Commerce site.

The e-Customer behaviour model can be useful both for navigational prediction (to predict future moves of a user and pre-fetch objects in order to improve performance) and workload prediction (to capture workload parameters to be used as input to a resource predictive model). By solving the set of linear equations that can be derived from the CBMG, the following metrics can be derived, including: average number of visits per state, average number of sale transactions per visit (i.e. buy to visit ratio), the average session length, the average number of requests submitted to the e-Commerce server per session etc.

III. E-Customer's Online Behaviour Viewed as a Stochastic Process

The randomness of the e-Customer's access to a particular e-Commerce Web site, and the unpredictability of invoking various specific functions (e.g. Browse, Search, Add to Cart, Select, Pay, etc.) during the online sessions, naturally imposes the idea of treating the whole interaction between e-Customers and e-Commerce Web site as being a stochastic process. Within this section we demonstrate the stochastic nature of the e-Customer's online behaviour in a rather informal manner.

Formally, a stochastic process is a family of random variables $\{X(t),\ t \in T\}$, defined on the same probability space, taking on values from the state space Ω, and being indexed by the parameter t, $\left(t \in T,\ T = [0,\infty)\right)$, which usually plays the role of time [3]. Each stochastic process is based upon the realization of a random experiment, which is a repeatable process whose outcome is not certain (known, determined) *a priori*. Each random experiment consists of two components: a procedure and an observation [3]. In the context of the e-Commerce, the procedure of the random experiment could be defined as follows: an e-Customer makes an access to a particular e-Commerce site and performs an online session by invoking specific functions, visually depicted by a corresponding CBMG diagram. Many various observations can be defined for a given procedure within a random experiment. For instance, an observation could be defined as making an evidence of the e-Commerce function being invoked by each consecutive e-Customer's click, during the online session. For the previously defined procedure, other possible observations could be also the following ones: measuring the time elapsed form the beginning of the session until the first invocation of the Search function, measuring the total duration of the session, making an evidence of the first three invoked functions, counting the total number of invoked functions during the session, counting the frequency of the invoked Search function, finding out the average time while being in Search mode, observing the sequence of invoked functions etc.

An outcome of an experiment is any possible observation of that experiment. Implicit in the definition of an outcome is the notion that each outcome is distinguishable from every other outcome. As a result, the universal set of all possible outcomes could be defined, also known as a sample space *S*. The sample space of an experiment is the finest-grain, mutually exclusive, collectively exhaustive set of all possible outcomes [3]. In the context of the previously defined random experiment and having on mind the CBMG diagram, the sample space S is the set of all possible specific functions offered by a particular e-Commerce Web site, i.e.

$$S = \{\text{Entry, Browse, Search, Select, Add to Cart, Pay, Exit}\}$$

Obviously, the sample space S is a finite set consisting of $n = |S|$, $n \in \mathbb{N}$ elements.

An event is a set of outcomes of an experiment. In fact, each subset of the sample space S is an event. For instance the following events could be defined: E_1 = {a subset of all e-Commerce functions which can be invoked after invoking the Browse function}; E_2 = {a subset of all e-Commerce functions whose name begins with the letter 'A'}; E_3 = {a subset of all e-Commerce functions from where the function Exit can be invoked}; etc.

By assigning a particular measure of probability $P[.]$ to all possible (both elementary and complex) outcomes of the random experiment, a probability space is being constructed, i.e. a probability $0 \le P[A] \le 1$ is being assigned to each event $A \subset S$. For instance, under an assumption that the probability of invoking any of the available e-Commerce functions is equally possible, then each elementary outcome, i.e. each element of the sample space S can be assigned a probability equal to $\frac{1}{n}$. Identically, an adequate probability can be also assigned to any complex outcome, like the following ones: "the Search function has been invoked more than X, but less than Y times", or "the Browse and Add to Cart functions have been invoked during the online session" or "the Pay function has not been invoked at all".

As previously stated, the definition of the stochastic process relays on the notion of a random variable. The online session lasts for a certain time period. A certain time period also expires between any two consecutively invoked e-Commerce functions, i.e. the e-Customer resides in each of the identified states of the CBMG a certain amount of time. Thus, to each identified state, i.e. to each element of the sample space S, a real number can be assigned, meaning the time elapsed in that particular state. Moreover, a real number between 0 and 1 can be assigned to each identified state, as the probability that e-Customer will reside in that particular state at an arbitrary moment. On the other hand, a random variable consists of an experiment with a probability measure $P[.]$, defined on a sample space S, and a real function $X : S \to \mathbb{R}$, that assigns a real number to each outcome in the sample space S of the experiment [3]. In this context, a discrete random variable X, describing the residence of the e-Customer in a particular state within the CBMG, can be defined as follows:

$$X = (x_1,\ x_2,\ x_3,\ ...,\ x_i,\ ...,\ x_n) \tag{1}$$

The random variable X is a vector consisting of finite number of elements, which represent, respectively, the particular states $x_i \in S$; $i = 1,\ 2,\ ...,\ n$; being visited by the e-Customer during the online session.

The assignment of a real number is made in accordance with a corresponding probability mass function (pmf) of the random variable X, i.e.

$$p_X = (p_1,\ p_2,\ p_3,\ ...,\ p_i,\ ...,\ p_n) \tag{2}$$

The probability mass function p_X is also a vector comprised of a finite number of elements, which represent the probabilities of being in a particular state $x_i \in S$; $i = 1,\ 2,\ ...,\ n$; i.e.

$$p_i = P\{X = x_i\};\ i = 1,\ 2,\ ...,\ n;\ \text{where} \sum_{i=1}^{n} p_i = 1 \tag{3}$$

Finally, the stochastic process $\{X(t),\ t \in T,\ T = [0, \infty)\}$ can be defined as a family of random variables X_j; $j = 1,\ 2,\ ...,\ m$; that corresponds to concurrent online sessions of m e-Customers with a particular e-Commerce Web site, that occur in parallel during the observed time period $[0, \infty)$.

Each stochastic process can be graphically visualized by means of trajectories, i.e. sample paths, as a family of functions being time dependent. Each sample path corresponds to an on line session of a particular e-Customer, defining a trajectory along the state space. Observing the stochastic process eliminates the uncertainty of its evolution in time. For instance, let us suppose that within the observed time segment $[0,\ t_k)$ two concurrent online sessions have occurred, including these ones: 1-2-3-4-5-6-7; and 1-3-4-2-7, having on mind the CBMG, depicted on Fig. 1. The corresponding sample paths $X_1(t)$ and $X_2(t)$ of the underlying stochastic process are shown on Fig. 2. These sample paths correspond, respectively, to the following sequences of e-Commerce functions: Entry $\to$ Browse $\to$ Search $\to$ Select $\to$ Add to Cart $\to$ Pay $\to$ Exit; and Entry $\to$ Search $\to$ Select $\to$ Browse $\to$ Exit, invoked by two different, concurrent e-Customers, shopping online.

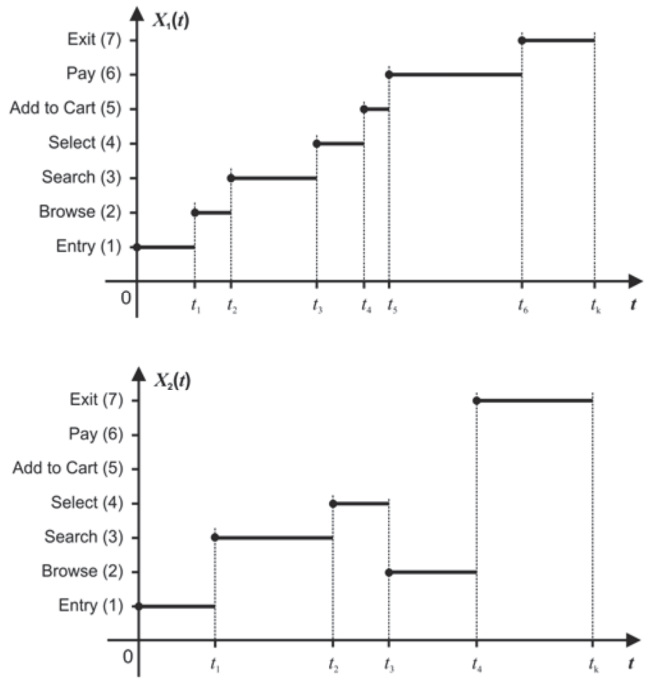

Figure 2: Two possible trajectories throughout the state space.

On Fig. 2 the discrete finite state space (i.e. the elements of Ω) is being depicted along the vertical axis, while the horizontal axis represents the continuous timeline. Thus, the e-Customers' online behaviour can be represented by a finite discrete state space and a continuous time space.

This way, the stochastic process can be only defined and visualized, which is impractical and useless for other purposes (e.g. for predictive modelling and evaluation). The complete characterization of the stochastic process needs a complete description of a vector consisting of a number of random variables, being extracted from the real process in each time instance, which is, by the way, an extremely difficult and practically impossible task. Therefore, the real-world processes are being approximated by special classes of stochastic processes, whose characterization is much simpler.

It can be shown that the e-Customer's online behaviour is, in fact, a Markov regenerative process, $\{Z(t),\ t \geq 0\}$, which is a generalization of the semi-Markov process [8], since the times between transitions are allowed to be random variables with general distributions, and since it does not matter what states the process has visited until reaching a specific state. This particular class of stochastic processes is a fundament (i.e. underlying process) of the class of Deterministic and Stochastic Petri Nets (DSPNs).

IV. DSPN MODEL OF THE E-CUSTOMER'S ONLINE SESSION

In computer science, the ordinary Petri Nets, known as Place/Transition Petri Nets (PTPNs) as being originally proposed, are widely recognized and utilized mathematical formalism and a graphical tool intended for modelling basic structural properties of complex systems, including: choices, iterations, concurrent execution, synchronization, blocking etc. Still, this particular class of Petri Nets can be utilized only for testing the functional and logical correctness of the modelled system, thus making possible the qualitative analysis solely. However, by adding temporal specifications of the transitions within the PTPNs, a quantitative analysis, i.e. modelling and evaluating of performance, reliability, availability, dependability and performability of the modelled system, have been made possible, too.

The class of Deterministic and Stochastic Petri Nets (DSPNs), originally proposed by Ajmone-Marsan & Chiola [4], is particularly suitable for modelling stochastic processes underlying e-Commerce online sessions, since it employs three types of temporal specifications for the transition firing delays: transitions with zero firing delays (immediate transitions), and two types of timed transitions, including: transitions with exponentially distributed firing delays, as well as transitions with deterministic (constant) firing delays. The deterministic firing delays are especially appreciated as a modelling element for activities having constant duration, e.g. timers.

The class of DSPNs is, in fact, an extension to Generalized Stochastic Petri Nets (GSPNs) by introducing transitions with deterministic firing delays, in addition to immediate and exponentially distributed timed transitions. Although DSPNs do not belong to the class of Markovian SPNs, their steady-state analysis is still possible, provided that at most one deterministically timed transition is being enabled at a time. Therefore, DSPNs do not allow more deterministic transitions to be concurrently enabled. This is not the case with the concurrent DSPNs, which can be analyzed by means of steady-state approximation. On the other hand, the subclass of extended DSPNs (eDSPNs) allows usage of expolynomially timed transitions, with the restriction that at most one non-exponentially timed transition (i.e. either one deterministic or one expolynomial transition) can be enabled at a time. Expolynomial distribution covers many well known probability distributions, like uniform or triangular distribution.

DSPNs employ continuous time scale for the underlying stochastic process, which is shown to be neither a Markov, nor a semi-Markov, but rather a Markov regenerative process [5]. A steady-state solution method for DSPNs is given in [4, 5, 6], whilst a transient (time-dependent) solution method is given in [5]. In addition, Ciardo & Lindemann [7] present a time- and space-efficient algorithm for computing steady-state solutions of DSPNs with both stochastic and structural extensions, which can deal with different execution policies associated with deterministic transitions, as well as use of reward structure to reduce memory requirements. Markov Regenerative Stochastic Petri Nets (MRSPNs), that are a superset of SPNs, GSPNs, ESPNs, and DSPNs, have been thoroughly elaborated in [8].

Bearing this in mind, Mitrevski et al. [9, 10, 11] have originally proposed a brand new framework for building up a predictive model, suitable for capacity planning, based on e-Customer's online behaviour and the usage of DSPNs as a modelling tool (Fig. 3).

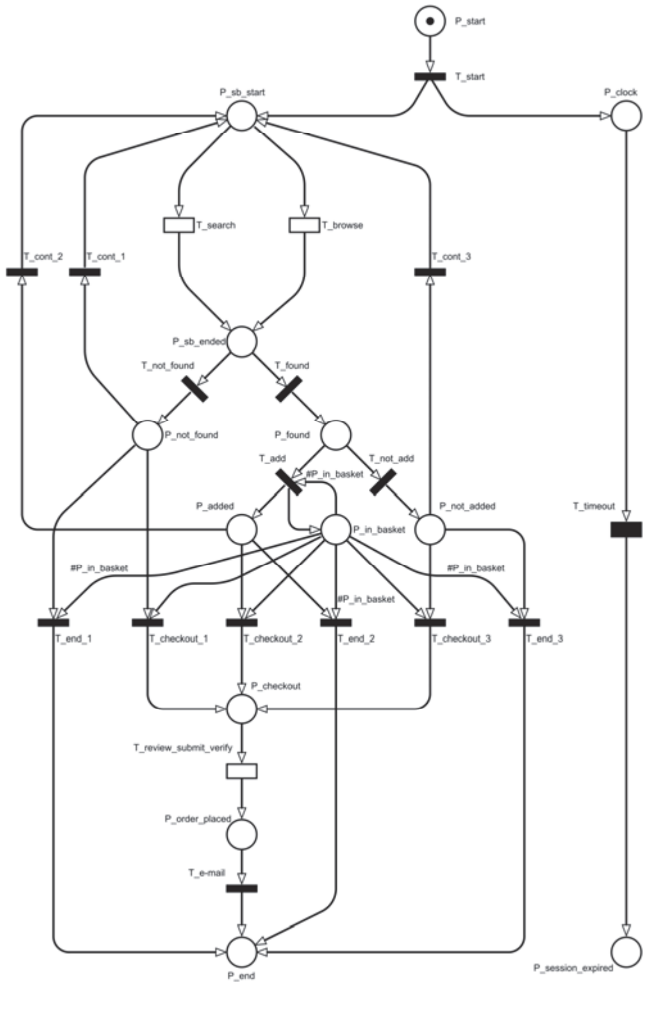

Figure 3: TimeNET v4.02 representation of the DSPN model for a single e-Customer, proposed by Mitrevski et al.

The proposed DSPN model (Fig. 3) captures well the e-Customer's online behaviour in a typical e-Commerce surrounding and reflects the specific business logic structure of a particular e-Commerce Web site, since it has been derived from the corresponding CBMG diagram. In addition, DSPN's building blocks truly depicts the stochastic nature of the real-world interaction, thus ensuring credibility and consistency of the results that are going to be obtained by solving the model itself. By finding the steady-state solution of the proposed DSPN model, the average time spent by an e-Customer during the online session, as well as the average number of customers in the system, can be calculated [7]. By means of the transient (time dependent) analysis, general service time can be estimated [5].

V. Software Tools for solving DSPN Models by Simulation

Modelling and evaluation of the stochastic behaviour of realistic models of dynamic systems can include utilization of various classes of Stochastic Petri Nets (SPNs), including DSPNs. As complexity of models grows, it might be very difficult, if not entirely impossible, to solve the model (the underlying stochastic process) analytically, meaning that a closed-form solution can not be (easily) derived. In that case, the only way to solve the model is to obtain a numerical solution through computer simulation software, having on mind its limitations. Such software environments are needed to simplify both model specification and modification, and to provide automated, time- and space-efficient quantitative analysis.

DSPNs belong to the class of non-Markovian SPNs, because they include, besides exponentially and immediate firing delays, also deterministic timed transitions. In addition, the stochastic process underlying DSPNs is not a pure Markov process. Consequently, DSPN models can not be solved by software simulation packages intended for solving the so-called Markovian SPNs, whose underlying stochastic process (i.e. the time evolution of the marking process) is a continuous-time Markov chain (CTMC), assuming the strong existence of the Markov property.

Currently, two software environments dedicated to solving DSPNs exist, including DSPNexpress 2000 (as a successor of the DSPNexpress v1.5) [12], and TimeNET v4.02 [13].

Novel innovative features of DSPNexpress 2000 constitute an efficient numerical method for transient analysis of DSPNs with and without concurrent deterministic transitions, as well as an effective numerical method for steady-state analysis of DSPNs with concurrent deterministic transitions.

TimeNET v4.02 (Timed Petri Net Evaluation Tool) is a software package intended for modelling and evaluation of SPNs in which the firing times of the transitions may be immediate, exponentially distributed, deterministic, or more generally distributed (so-called expolynomial distributions). Consequently, it offers numerical analysis methods for GSPNs, DSPNs, extended DSPNs, Stochastic Coloured PNs (SCPNs) and an approximation method for concurrent DSPNs. Besides this, simulation of arbitrary SPNs is possible, too. For all supported classes of SPNs, steady-state and transient analyses can be carried out. The stationary analysis is based on Markov regenerative theory, assuming construction and solving of an embedded Markov chain (EMC). The transient analysis is based on the method of supplementary variables. Reward measures can be specified, too, as well as various expolynomial probability distributions. TimeNET also allows structural analysis and verification, token game animation, identification of place invariants, as well as simple/advanced performance analysis.

Conclusion

The paper points out the necessity of building predictive models for capacity planning of e-Commerce Web site infrastructure. The key starting point is the process of observing, analyzing, graphing, and modelling e-Customer's online behaviour. Several models presenting the online interaction, including the CBMG, have been proposed so far. Having on mind the stochastic nature of online interaction, the modelling framework being proposed by Mitrevski et al., based on the use of the class of DSPNs, represents a solid basis for obtaining relevant performance indicators, still remaining a platform for further research in this area. The software implementation of the DSPN model in TimeNET v4.02 offers great potentials for solving the underlying stochastic process by simulation.

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